The Flagellation of Christ, c. 1458-60. Piero della Francesca. Tempera. 59 x 81.5 cm. Urbino, Galleria Nazionale delle Marche.
Albrecht Durer. **Untitled** (Artist using a glass to take a portrait). From *Underweysung der Messung mit dem Zirkel und Rüchtscheyt*, 1st Ed, 1525. Woodcut print.
Albrecht Durer, Untitled (Two draughtsmen plotting points for the drawing of a lute in foreshortening) From *Underweysung der Messung mit dem Zirkel und Riichtscheyt*, 1st Ed., 1525. Woodcut print.
Reference

Planar Geometric Projections

Parallel
- Orthographic
  - Multiview Orthographic
  - Axonometric
- Oblique
- Perspective
  - One-point
  - Two-point
  - Three-point
Orthographic Projections

Projectors are perpendicular to the image plane

Object faces are parallel to the image plane

Axonometric Projections

Projectors are perpendicular to the image plane

Object faces are not parallel to the image plane

Figure 2.4 The three types of axonometric projections

Oblique Projections

Projectors are parallel but not perpendicular to the image plane

Object faces are parallel to the image plane
Perspective Projection

Projectors are not parallel but converge on a single focal point (eye, camera)

The concept of the picture plane may be better understood by looking through a window or other transparent plane from a fixed viewpoint. Your lines of sight, the multitude of straight lines leading from your eye to the subject, will all intersect this plane. Therefore, if you were to reach out with a grease pencil and draw the image of the subject on this plane you would be "tracing out" the infinite number of points of intersection of sight rays and plane. The result would be that you would have "transferred" a real three-dimensional object to a two-dimensional plane.
Rays of light travel from the object, through the picture plane, and to the viewer's eye. This is the basis for graphical perspective.
Computer Graphics
Perspective Image Generation
Standard Computer Graphics Pipeline

- Model
  - Camera
    - Perspective Transform
    - Raster Operations
    - Image Storage
  - Display
Standard Computer Graphics Pipeline
The camera location, view direction, and frustum must be defined relative to the object.
Eye Coordinate System

The model is described in a right handed coordinate system.
Note the eye coordinate system is a left-handed coordinate system.
Left Handed and Right Handed Coordinate Systems

Left Handed

Right Handed

Thumbs up for understanding!
Simple Perspective Transformation

Plane

Point On Object

Diagram:

\[ P(x_e, y_e, z_e) \]

\[ P'(x_s, y_s) \]

Picture Plane

S

S

\[ X_e \]

\[ Y_e \]

\[ Z_e = 0 \]

\[ Z_e = D \]
Simple Perspective Transformation

Elevation drawn in the $Y_e, Z_e$ plane.
Simple Perspective Transformation

\[
\begin{align*}
x_s &= \frac{x_e}{D} \quad & y_s &= \frac{y_e}{D} \\
&= \frac{x_e}{z_e} \quad & y_s &= \frac{y_e}{z_e} \\
&= \frac{Dx_e}{z_e} \quad & y_s &= \frac{Dy_e}{z_e}
\end{align*}
\]

To convert to a dimensionless fraction, can divide by the window size \( S \).

\[
\begin{align*}
x_s &= \frac{Dx_e}{Sz_e} \\
y_s &= \frac{Dy_e}{Sz_e}
\end{align*}
\]
Perspective Projection

Eye coordinate system

Picture Plane

Object

Plan or elevation view
Pinhole Camera

Note that the entire image through the pinhole is totally in focus on a single image plane.
Ibn al-Haitham (Al-Hazen)

Credited with the having built the first camera obscura in the 10th Century.
Camera Obscura
Brunelleschi’s Perspective Experiment

• How do you draw a perspective image?

• How do you know it is correct?
Brunelleschi’s first experiment: overhead view of Florence Cathedral and the Baptistery with indication of the position of the observer inside the central portal and his two possible angles of vision.
Plan of the environs of the cathedral and the Baptistry of San Giovanni. The broken lines correspond to the two possible angles of vision. A. Volta dei Pecori (corner of the sheep market); B. Canto alla Paglia (corner of the straw market); C. Misericordia; D. Saint Zenobius column.
Ghiberti’s Doors
16 Brunelleschi's first experiment: how the tavolettia was used.
Illustration x–1: Reconstruction of a ninety-degree visual angle from the portal of the Duomo toward the Baptistery. Site-plan from Sgrilli’s Descrizione. . .
Brunelleschi’s Experiment
Brunelleschi’s Experiment
Brunelleschi’s Experiment
Brunelleschi’s Experiment
Description of Homework Assignment #1
End...