Visual Imaging in the Electronic Age

Perspective Transformations

Lecture # 3
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We have seen how a pinhole camera (or camera obscura) works.

We have also seen Brunelleschi’s experiment with his perspective panels.

Both are obviously two dimensional and exist on a planar (flat) surface.
Perspective Depth

• We now know how to mathematically (and geometrically) construct the perspective image. (e.g., \(x_s, y_s\))
• But how do we find the third dimension \(z_s\)?
How a Camera Lens Works

• Rays which pass through the focal point are bent to leave the lens perpendicular to its principal axis.
Railroad Tracks in Perspective

$z_{ea} + \alpha(z_{eb} - z_{ea})$

$Y_e$

$x_e, y_e, z_e$

$Z_e$
How a Camera Lens Works

- Rays which pass through the focal point are bent to leave the lens perpendicular to its principal axis.
- Rays which are perpendicular to the principal axis of the lens are bent through the focal point.
How a Camera Lens Works

\[ P(x, y, z) \]

\[ P'(x_p, y_p, z_p) \]

\[ -y_p \]

\[ \text{B}' \]

\[ \text{B} \]

\[ \text{A}' \]

\[ \text{A} \]

\[ \text{P}(x_e, y_e, z_e) \]

\[ \text{X}_e \]

\[ \text{Y}_e \]

\[ \text{Z}_e \]

\[-z_p - 2f\]

\[ f \]

\[ f \]
How a Camera Lens Works

Using triangles A and A':
\[
\frac{-y_p}{f} = \frac{y_e}{z_e}, \quad \frac{-x_p}{f} = \frac{x_e}{z_e}
\]

Using triangles B and B':
\[
\frac{y_e}{f} = \frac{-y_p}{-z_p - 2f}
\]

\[
\therefore x_p = -f \frac{x_e}{z_e}, \quad y_p = -f \frac{y_e}{z_e}
\]
How a Camera Lens Works

We know that:

\[- \frac{y_p}{y_e} = \frac{f}{z_e}, \quad - \frac{y_p}{y_e} = - \frac{z_p - 2f}{f}\]

\[\therefore - \frac{z_p - 2f}{f} = \frac{f}{z_e}, \quad z_p + 2f = - \frac{f^2}{z_e}\]

\[z_p = -2f - \frac{f^2}{z_e}\]
How a Camera Lens Works
Camera
Film
Plane
Camera
Film
Plane
In photography, the near focus or distance for a given aperture, the scales on a lens barrel, and focal distance oppose each other. If you then increase the depth of field with the camera has a hyperfocal focus at 18 feet, the depth of field will increase to infinity. For
Perspective Projections in
Computer Graphics

Eye coordinate system

Plan or elevation view
Object Space

(One point perspective)
Image Space

(One point perspective)
3D Perspective Image
(x & y coordinates in virtual image space)

\[
x_p = -f \frac{x_e}{z_e}
\]
\[
y_p = -f \frac{y_e}{z_e}
\]

\[
x_p = \text{constant} \frac{x_e}{z_e}
\]
\[
y_p = \text{constant} \frac{y_e}{z_e}
\]
3D Perspective Image (z depth coordinate in virtual image space)

\[ z_p = -2f - \frac{f^2}{z_e} \]

\[ z_{p_i} = \text{constant}_1 + \text{constant}_2 \frac{1}{z_e} \]
Equivalent Distorted Geometries in Virtual Image Space

Plan or elevation view
Mapping a Viewing Frustum to a Standard Viewbox

Frustum of vision

Screen coordinate system

\[ z_e = D \]

\[ z_e = F \]
Viewing Frustum - User Parameters

Location and Orientation (vectors):
- \textit{Eye} location of eye point
- \textit{At} a point along view direction vector
- \textit{Up} a vector defining the Y axis on the hither plane

Shape (scalars):
- \textit{fov} field of view angle
- \textit{aspect} view plane aspect ratio
- \textit{hither} perpendicular distance to hither plane
- \textit{yon} perpendicular distance to yon plane

\textbf{Aspect} = \frac{\text{width}}{\text{height}}
Uses of Perspective in Enlarging Architectural Space (3D)
Palazzo Spada

Borromini
Imss, Firenze 2004 - 3 Scenografia prospettica
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Palazzo Spada
Palazzo Spada

Borromini
Palazzo Spada

Borromini
Probe Demo