Perspective Transformations

Visual Imaging in the Electronic Age

Donald P. Greenberg

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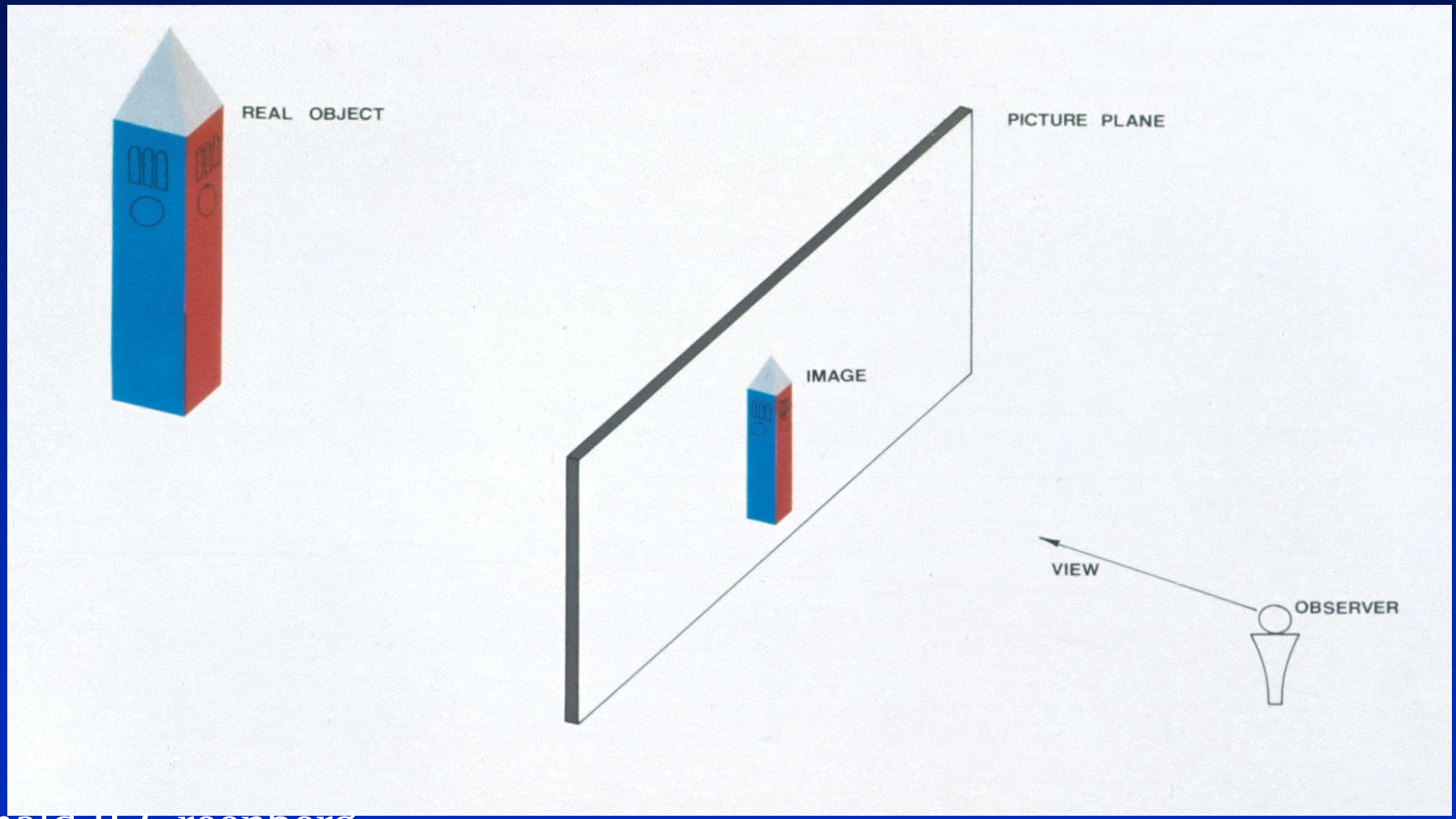
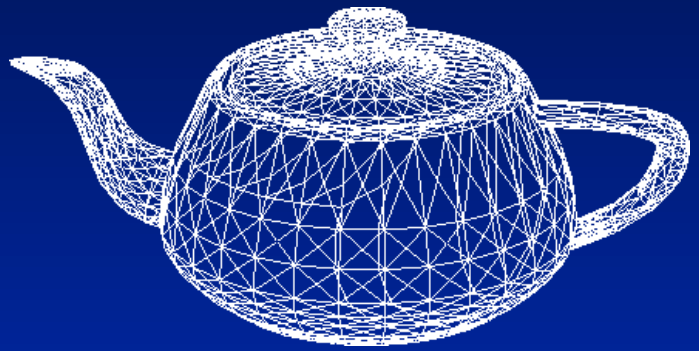
Lecture #3

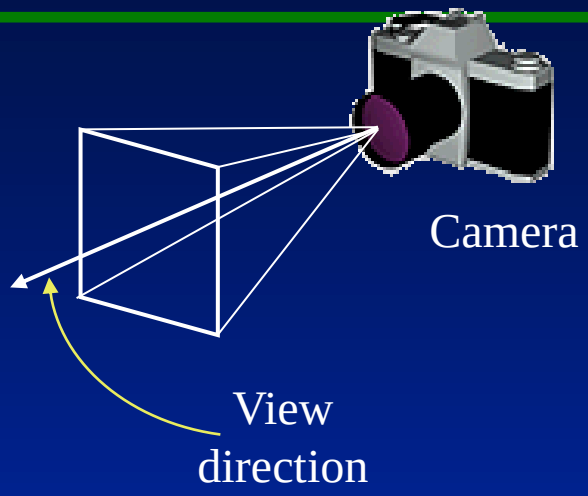
Perspective Image Generation



Camera Definition



Model



The camera location, view direction, and frustum must be defined relative to the object.

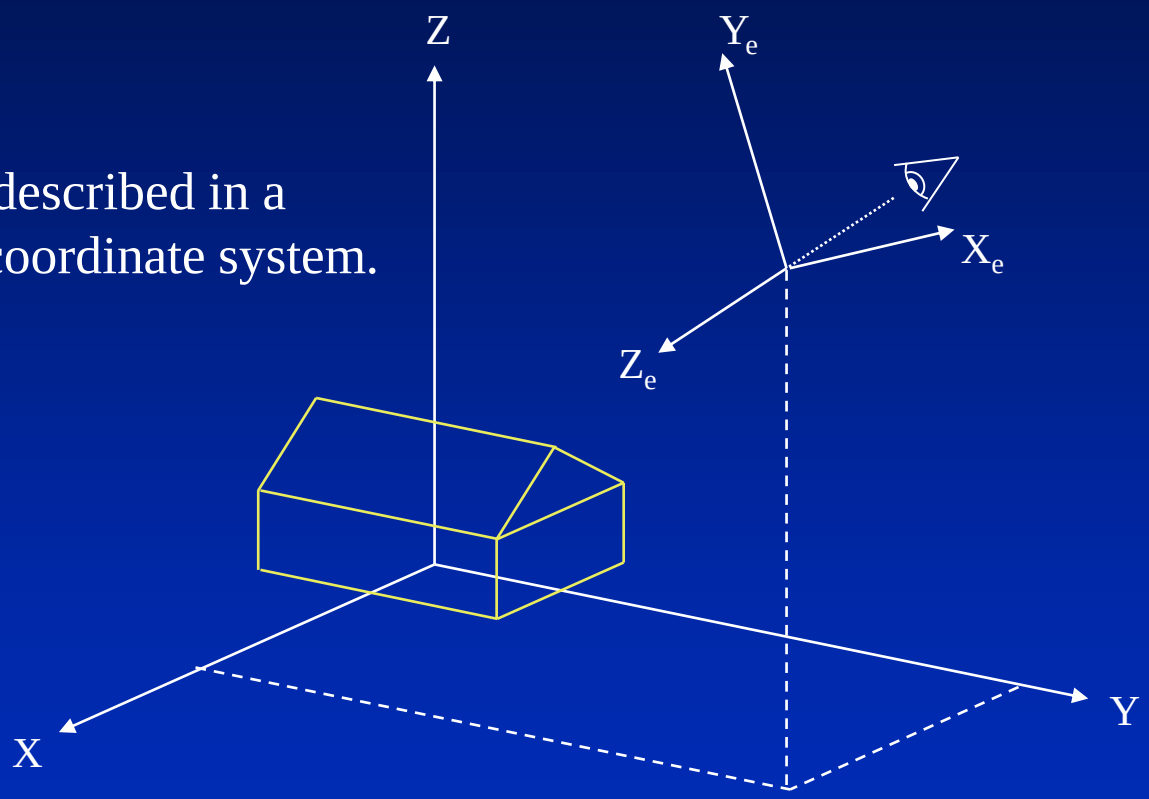
- We have seen how a pinhole camera (or camera obscura) works
- We have also seen Brunelleschi's experiment with his perspective panels
- Both pictures are obviously two dimensional and exist on a planar (flat) surface

Florence Baptistery

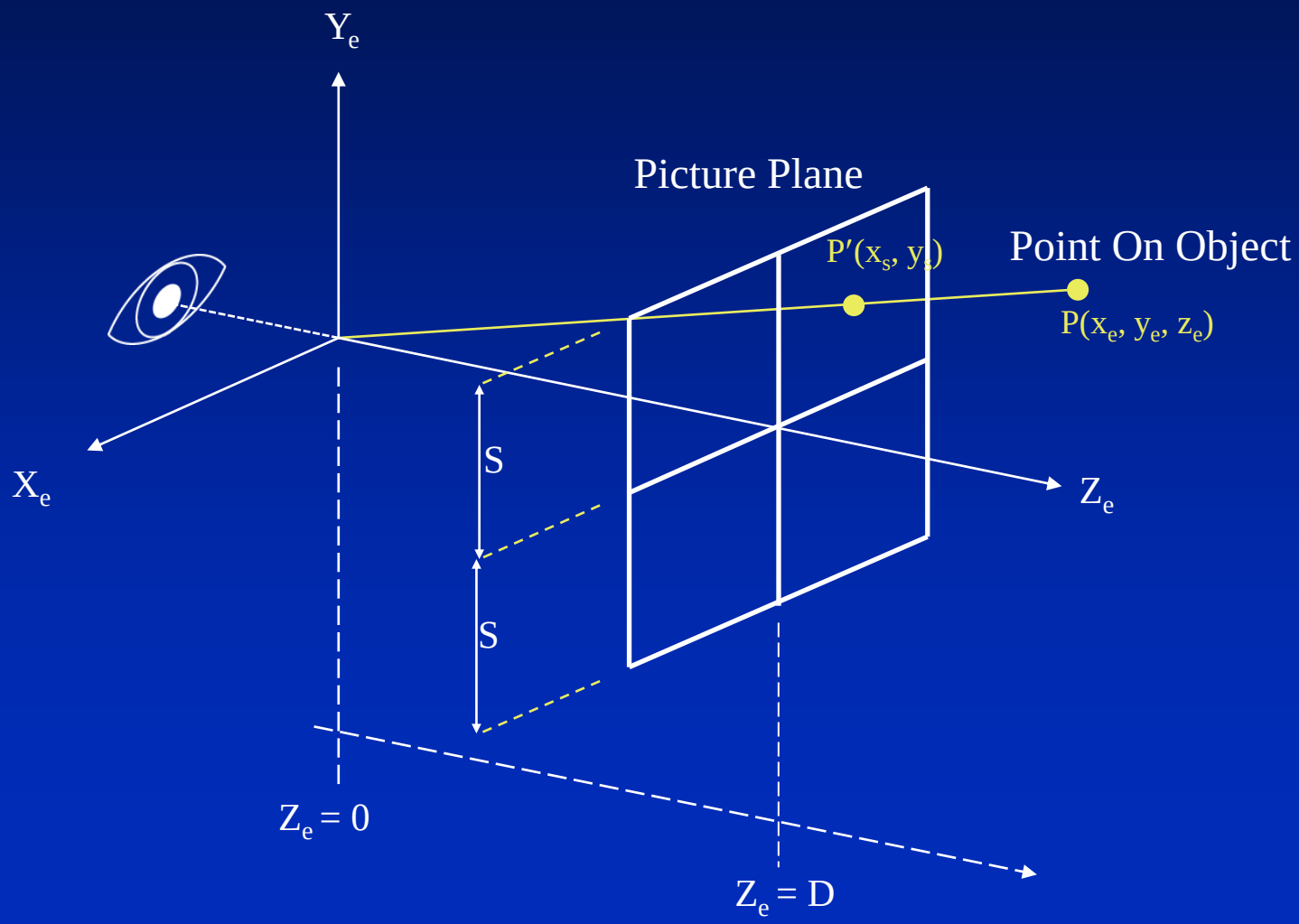
Brunelleschi's Experiment

Eye Coordinate System

The model is described in a right handed coordinate system.



Simple Perspective Transformation



Simple Perspective Transformation

$$\frac{x_s}{D} = \frac{x_e}{z_e}, \qquad \frac{y_s}{D} = \frac{y_e}{z_e}$$

$$x_s = \frac{Dx_e}{z_e}, \qquad y_s = \frac{Dy_e}{z_e}$$

To convert to a dimensionless fraction, can divide by the window size S.

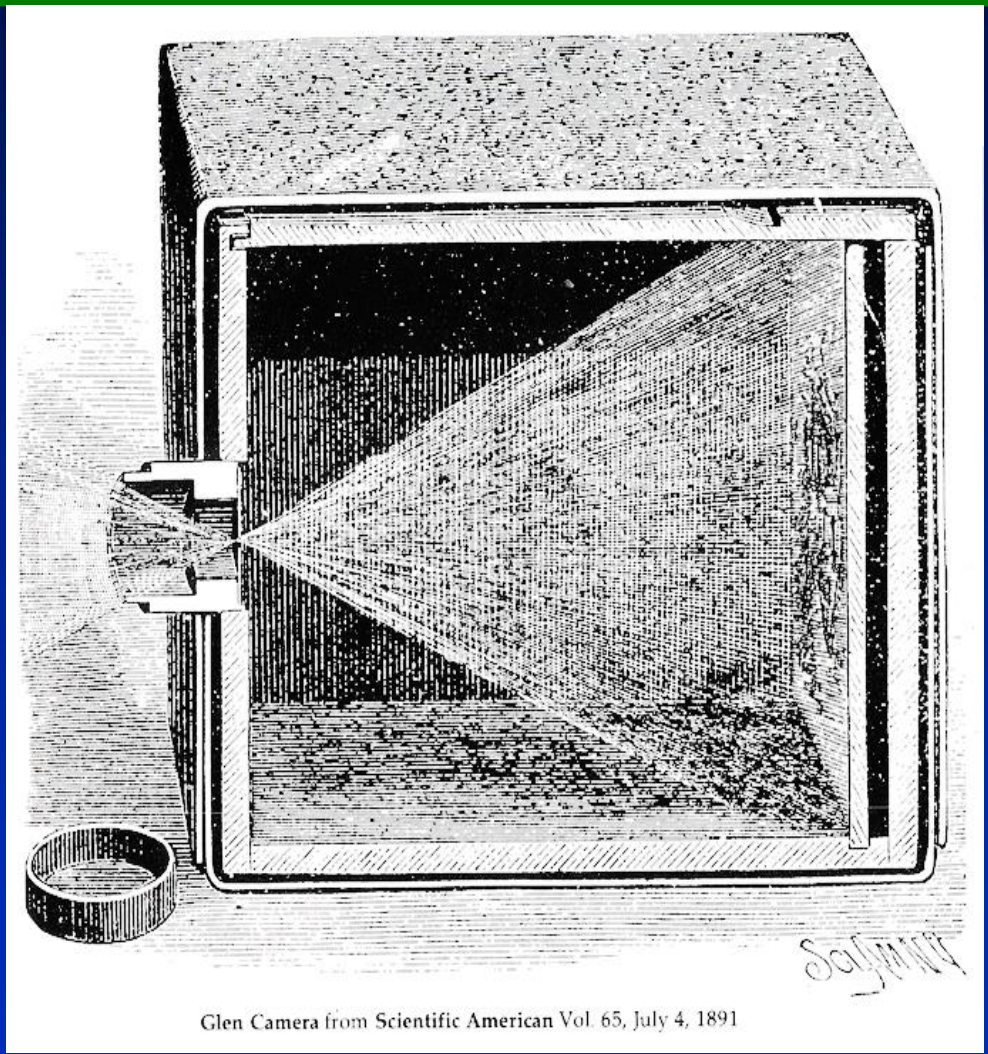
$$x_s = \frac{Dx_e}{Sz_e}, \qquad y_s = \frac{Dy_e}{Sz_e}$$

End of Review

Perspective Depth

- We now know how to mathematically (and geometrically) construct the perspective image. (e.g., x_s, y_s)
- But how do we find the third dimension (z_s)?

Pinhole Camera



Glen Camera from Scientific American Vol. 65, July 4, 1891

Note that the entire image through the pinhole is totally in focus on a single image plane.

Are camera images two-dimensional?

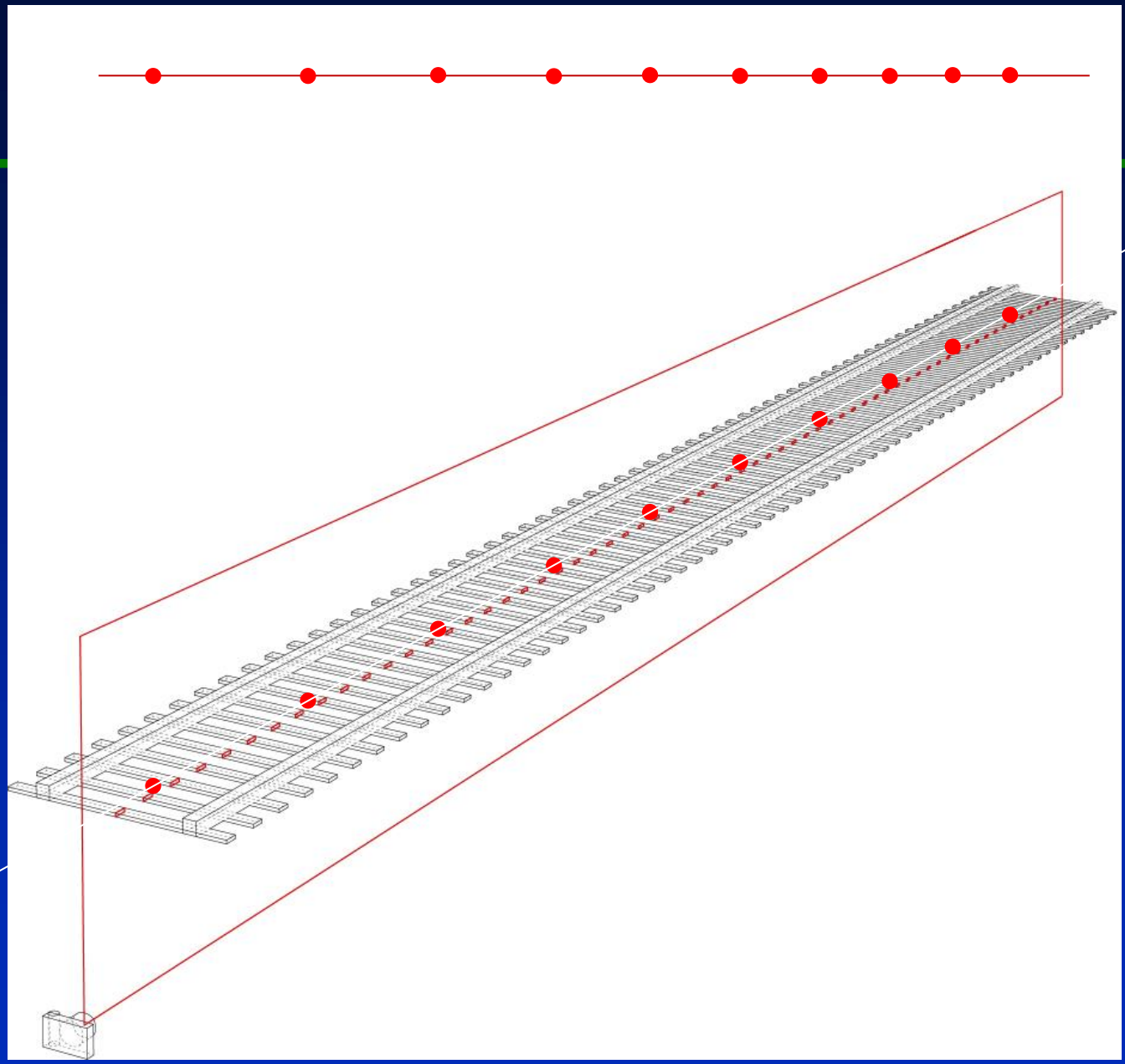
The image is recorded on a flat piece of film, or today, a digital sensor.

What is the difference between a pinhole camera and camera lens?

How a Camera Lens Works

- Rays which pass through the focal point are bent to leave the lens perpendicular to its principal axis.

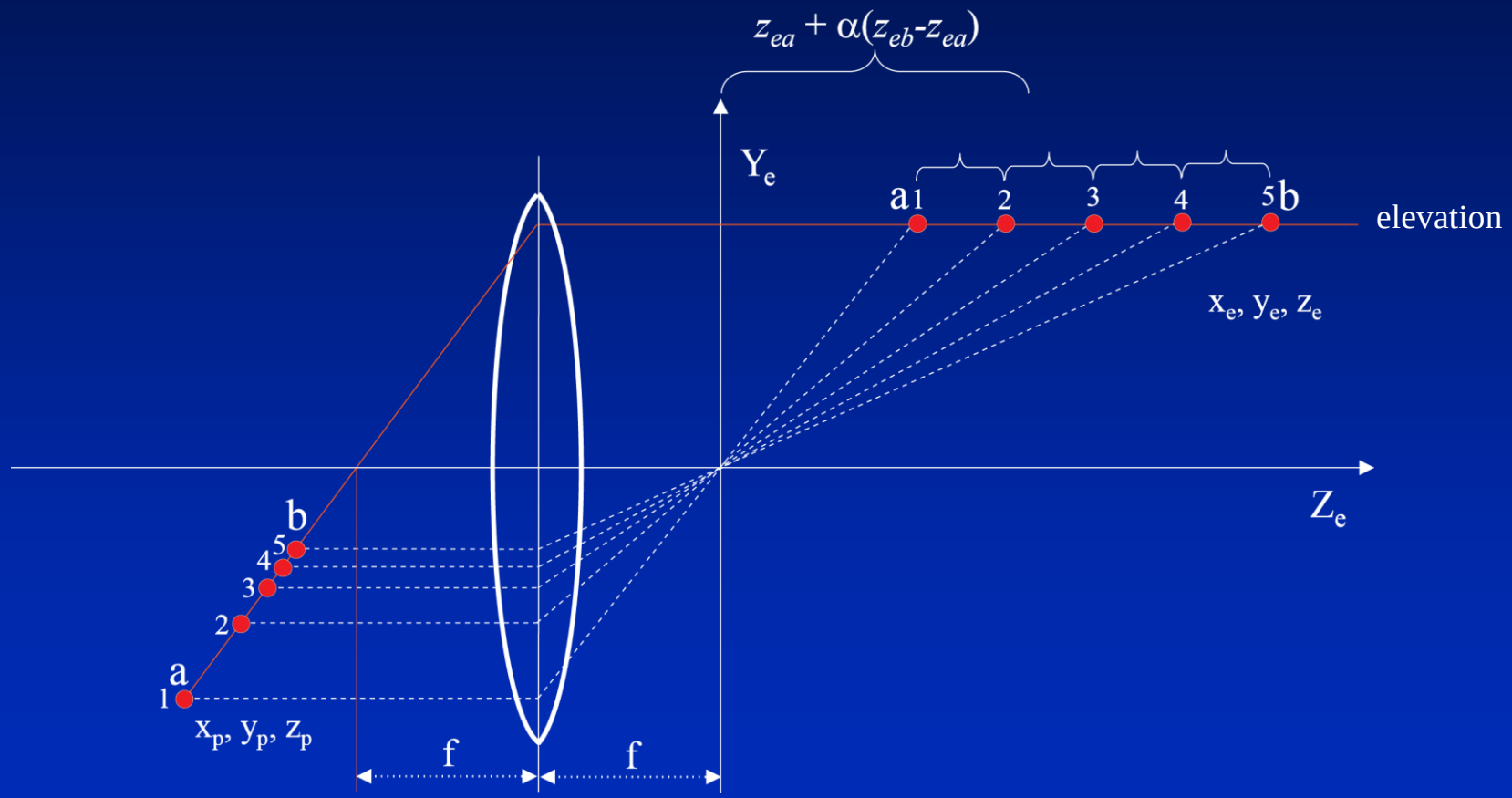
- Points equally spaced in a real environment are not equally spaced after perspective transformations.



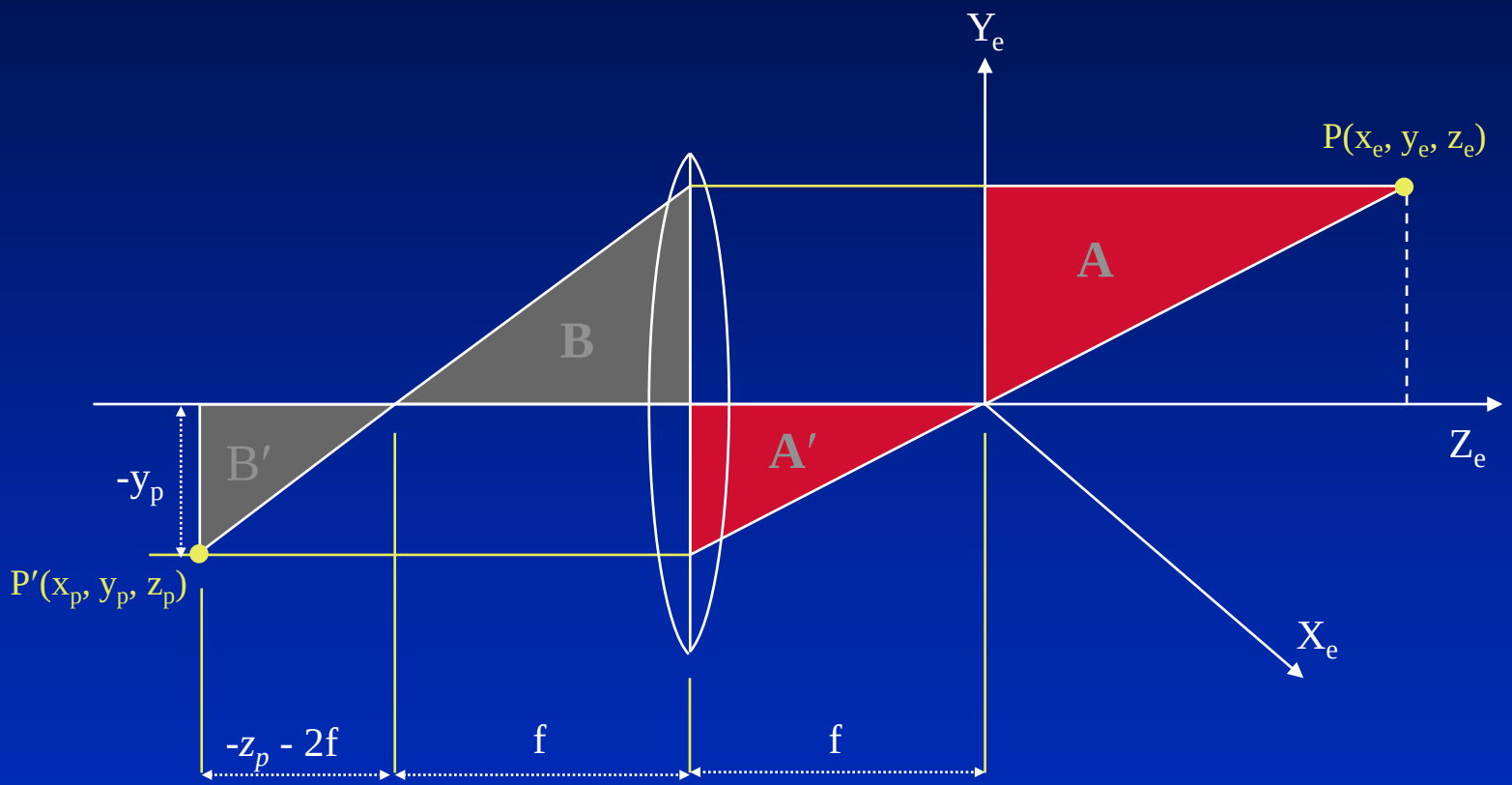
How a Camera Lens Works

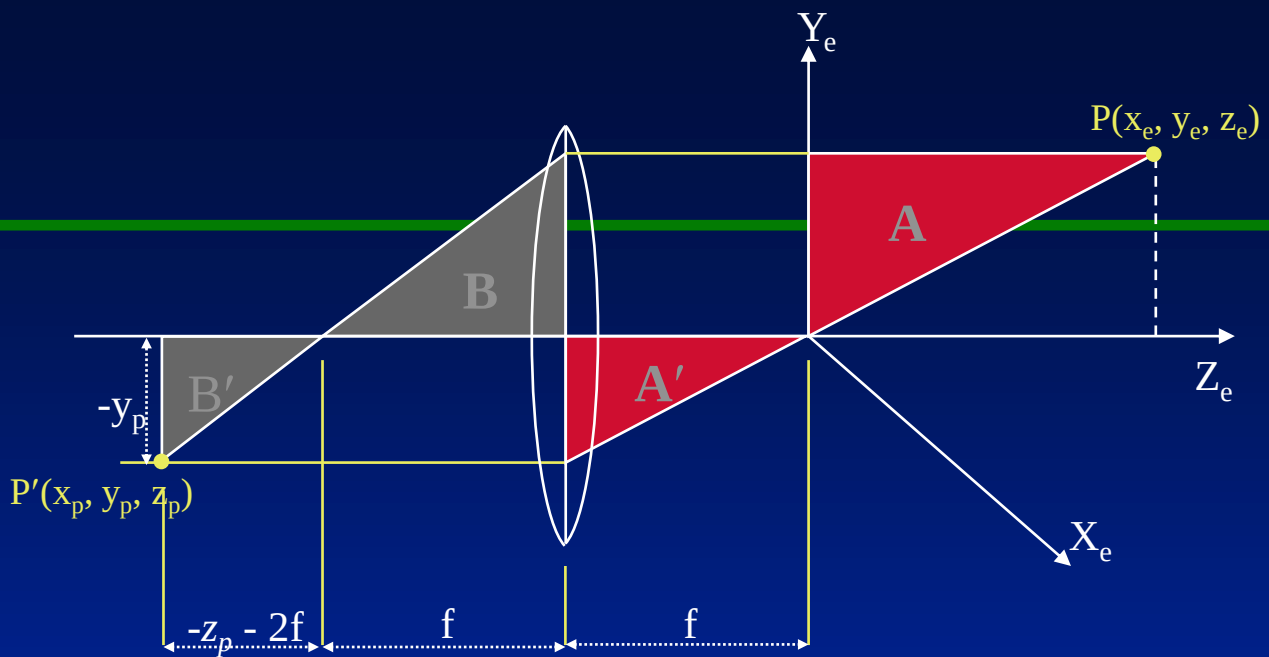
- Rays which pass through the focal point are bent to leave the lens perpendicular to its principal axis.
- Rays which are perpendicular to the principal axis of the lens are bent through the focal point.

Railroad Tracks in Perspective



How a Camera Lens Works





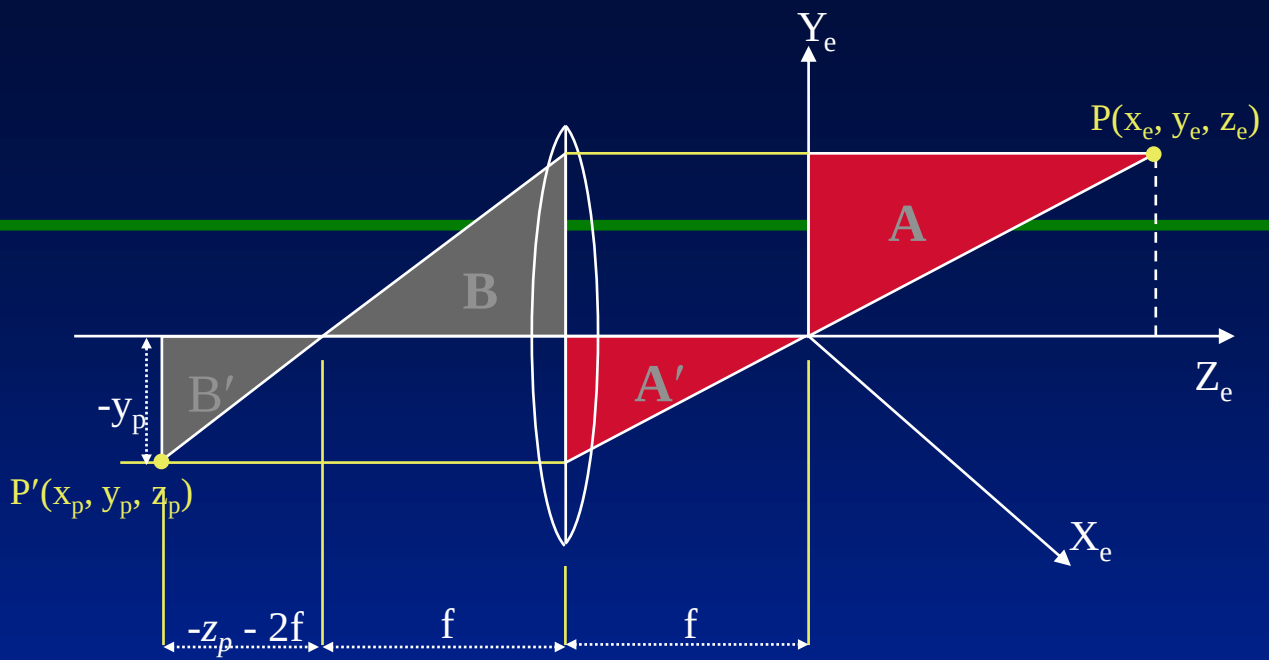
Using triangles A and A':

$$\frac{-y_p}{f} = \frac{y_e}{z_e}, \qquad \frac{-x_p}{f} = \frac{x_e}{z_e}$$

Using triangles B and B':

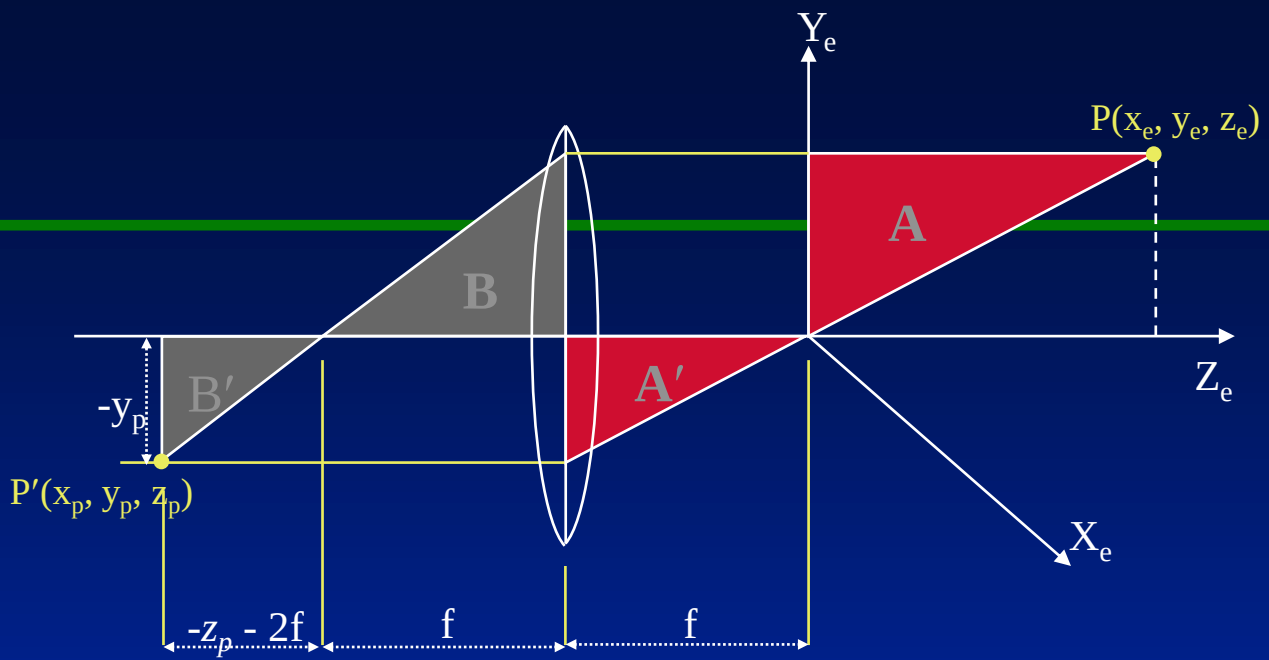
$$\frac{y_e}{f} = \frac{-y_p}{-z_p - 2f}$$

$$\therefore \quad x_p = -f\frac{x_e}{z_e}, \quad y_p = -f\frac{y_e}{z_e}$$



Using triangles A and A': $\frac{-y_p}{f} = \frac{y_e}{z_e}$

Using triangles B and B': $\frac{y_e}{f} = \frac{-y_p}{-z_p - 2f}$



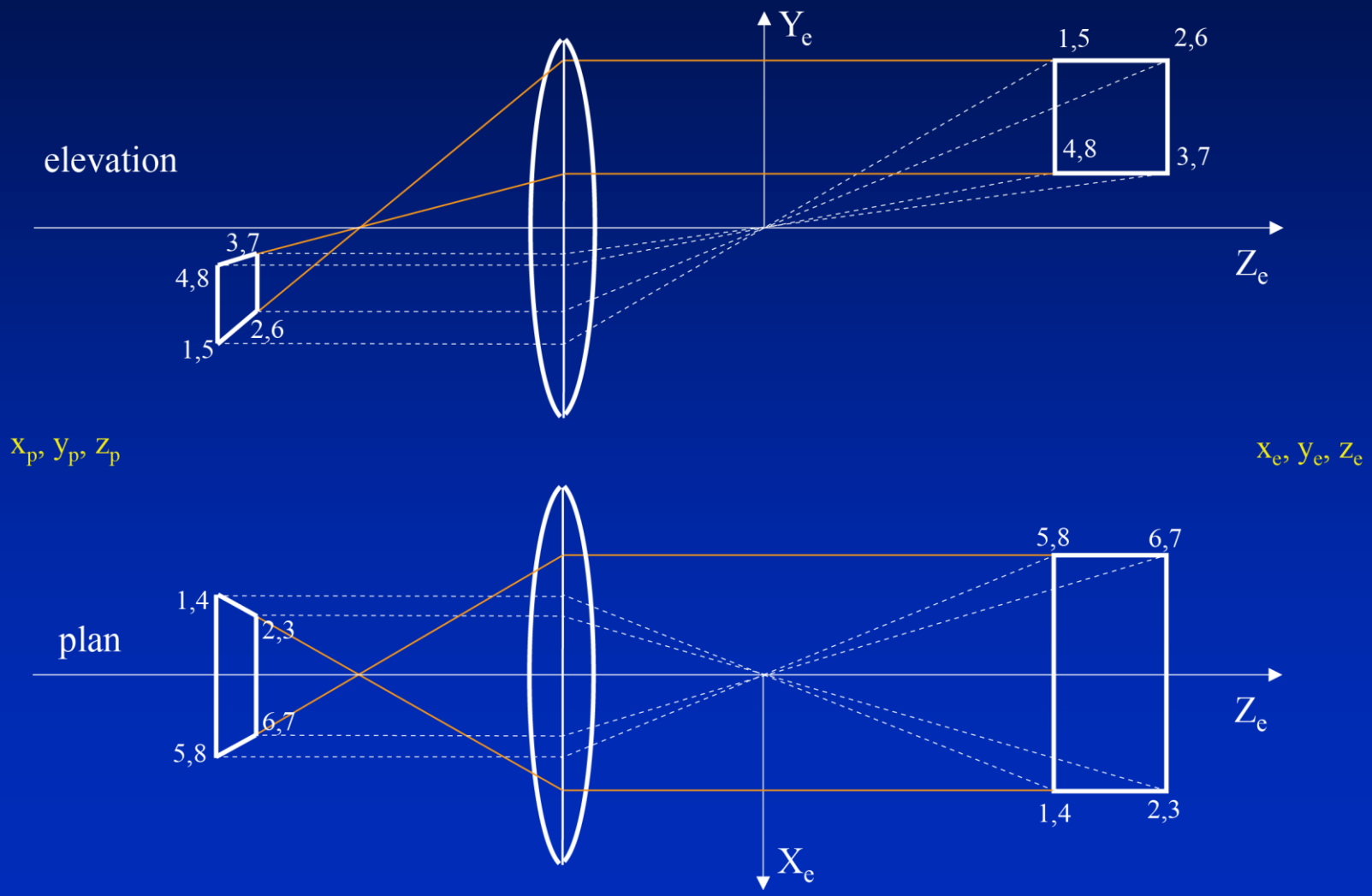
We know that:

$$\frac{-y_p}{y_e} = \frac{f}{z_e}, \quad \frac{-y_p}{y_e} = \frac{-z_p - 2f}{f}$$

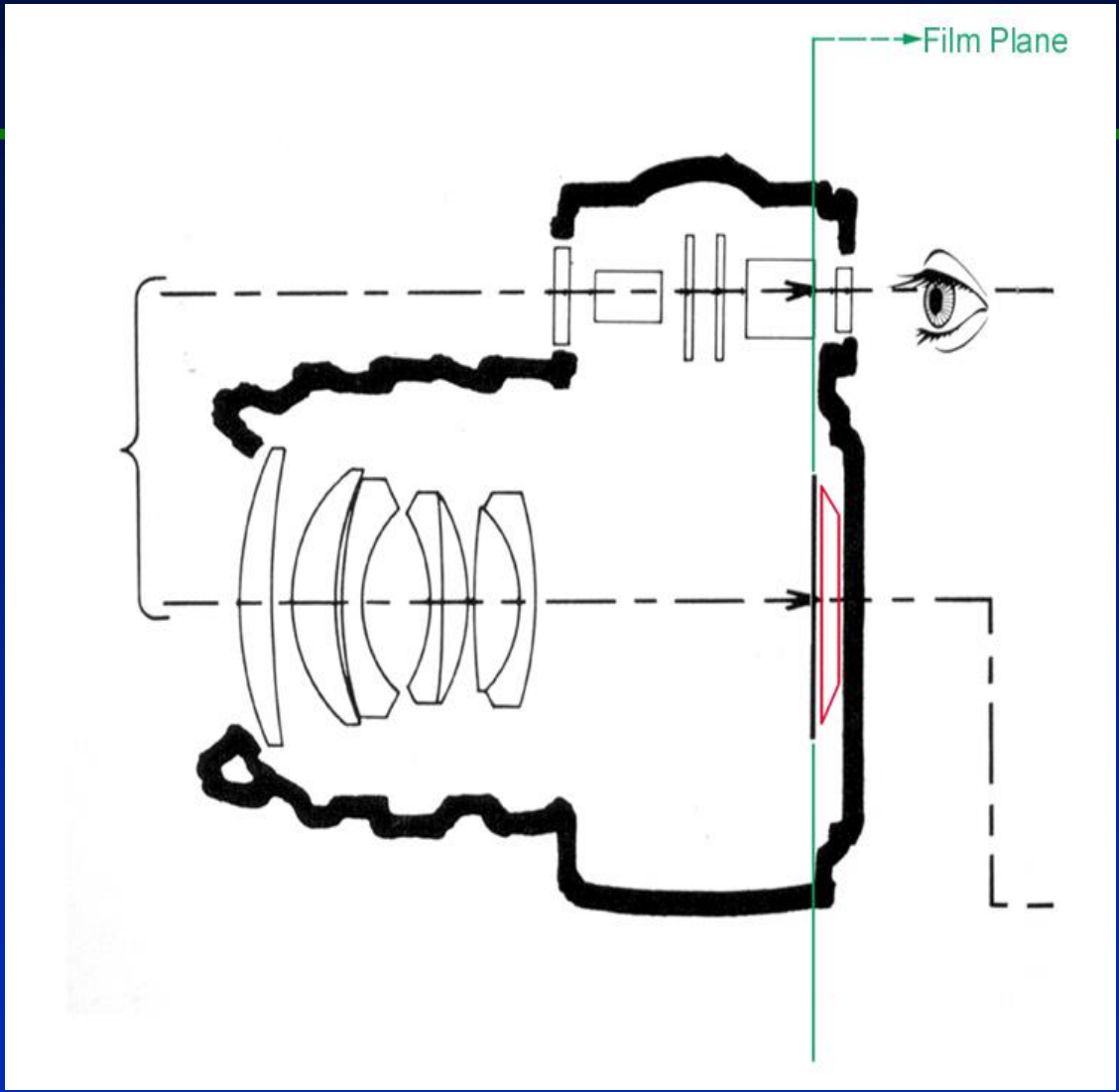
$$\therefore \quad \frac{-z_p - 2f}{f} = \frac{f}{z_e}, \quad z_p + 2f = \frac{-f^2}{z_e}$$

$$z_p = -2f - \frac{f^2}{z_e}$$

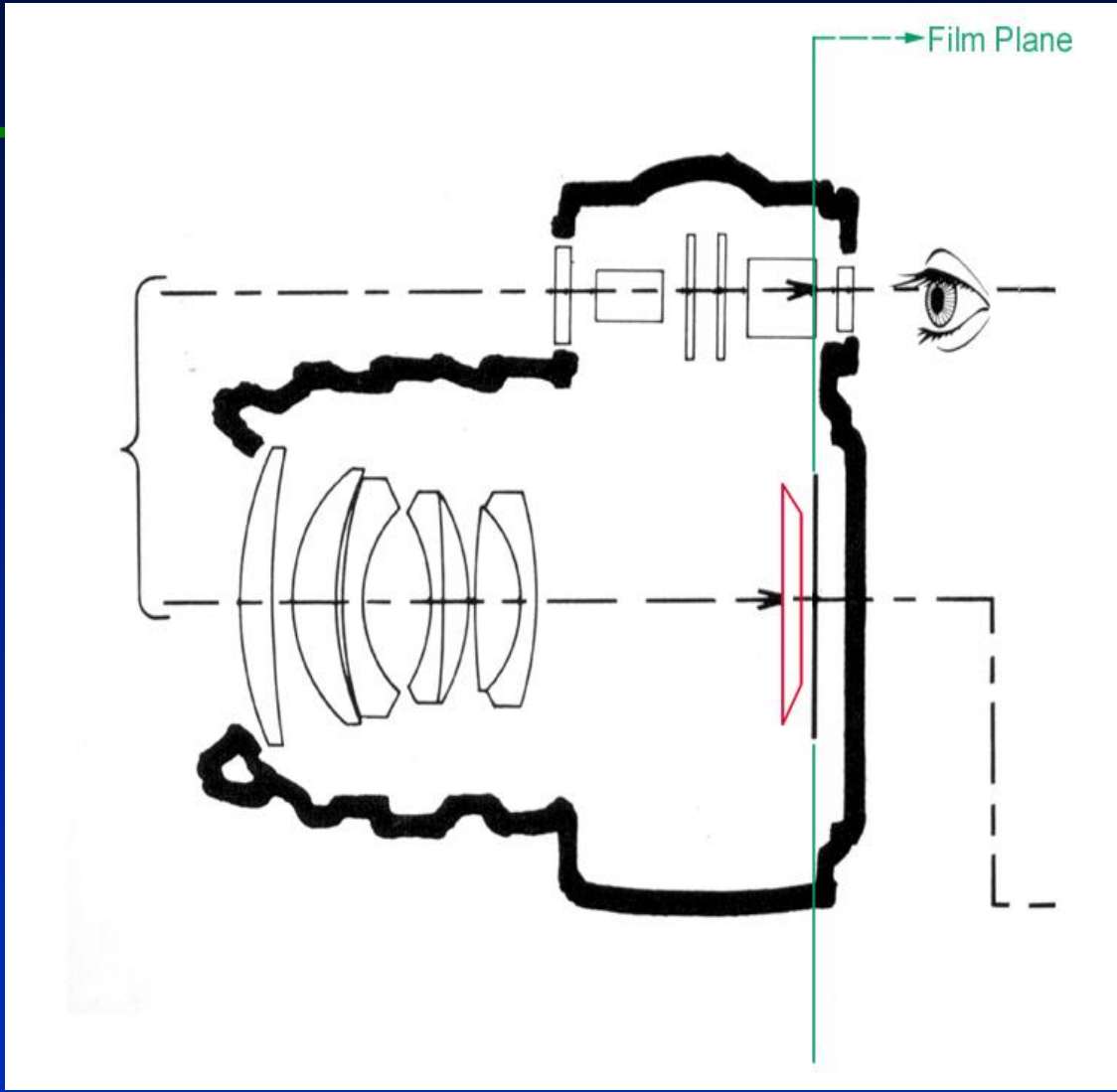
How a Camera Lens Works



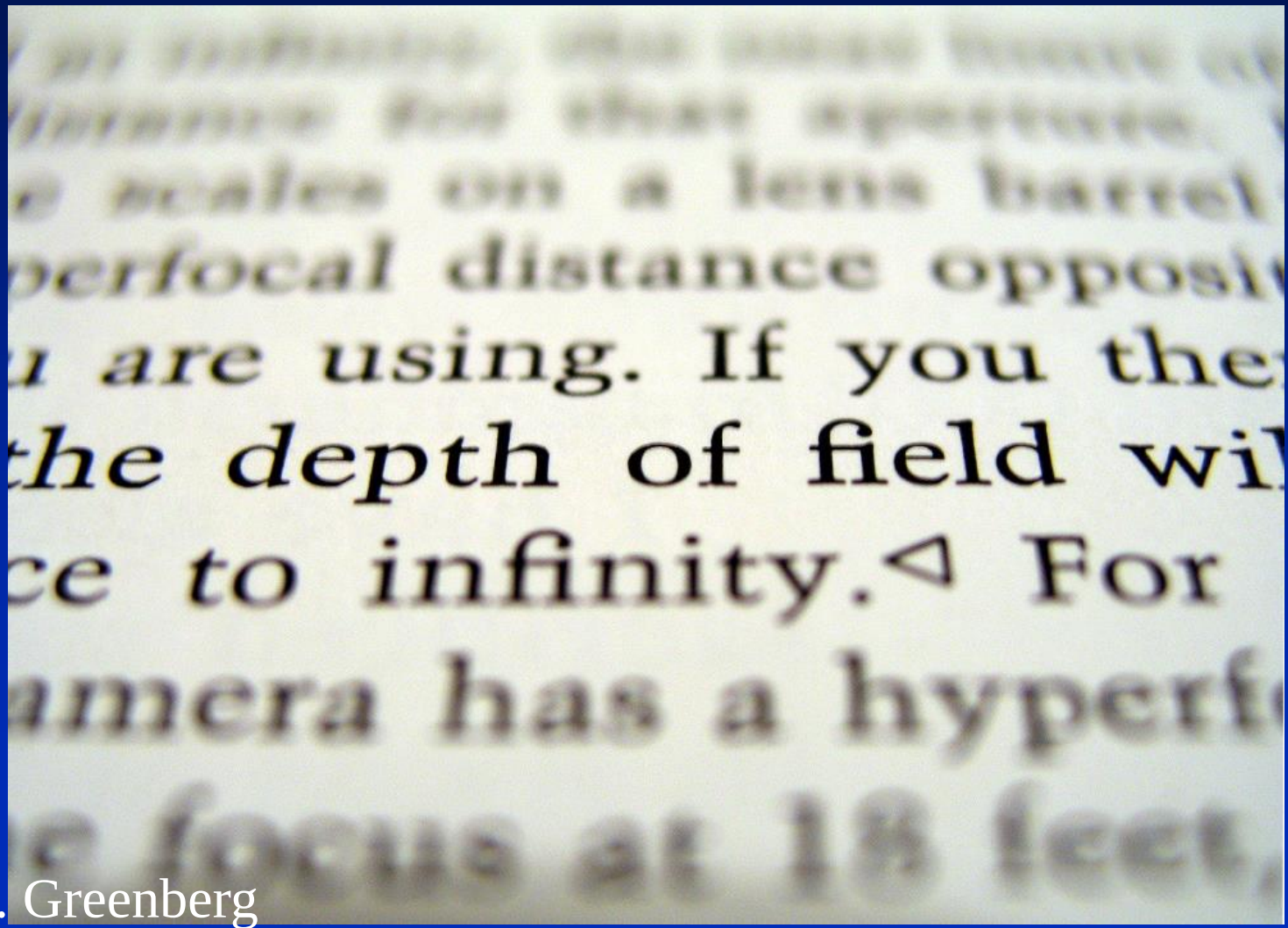
Camera Film Plane



Camera Film Plane

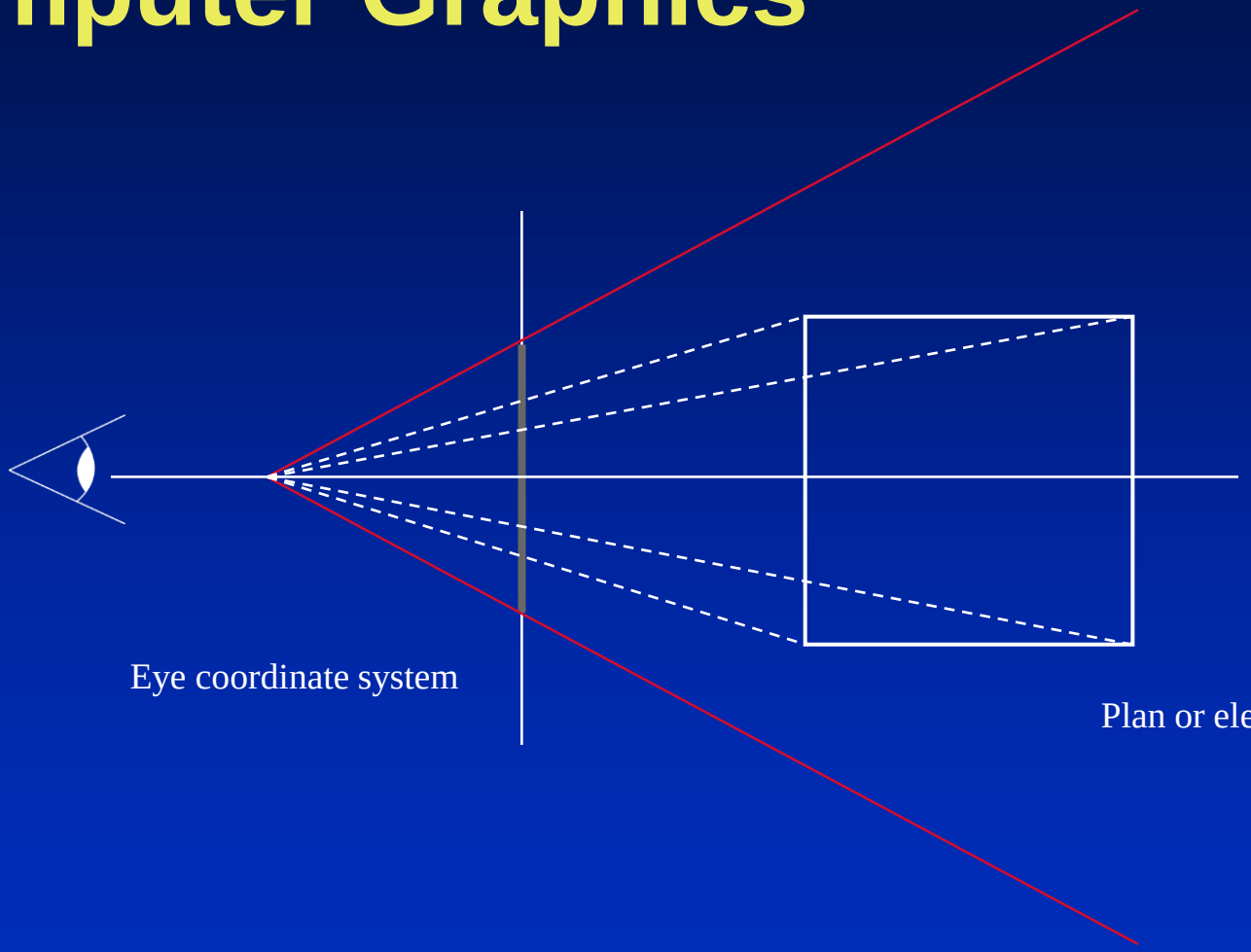



c
b
a

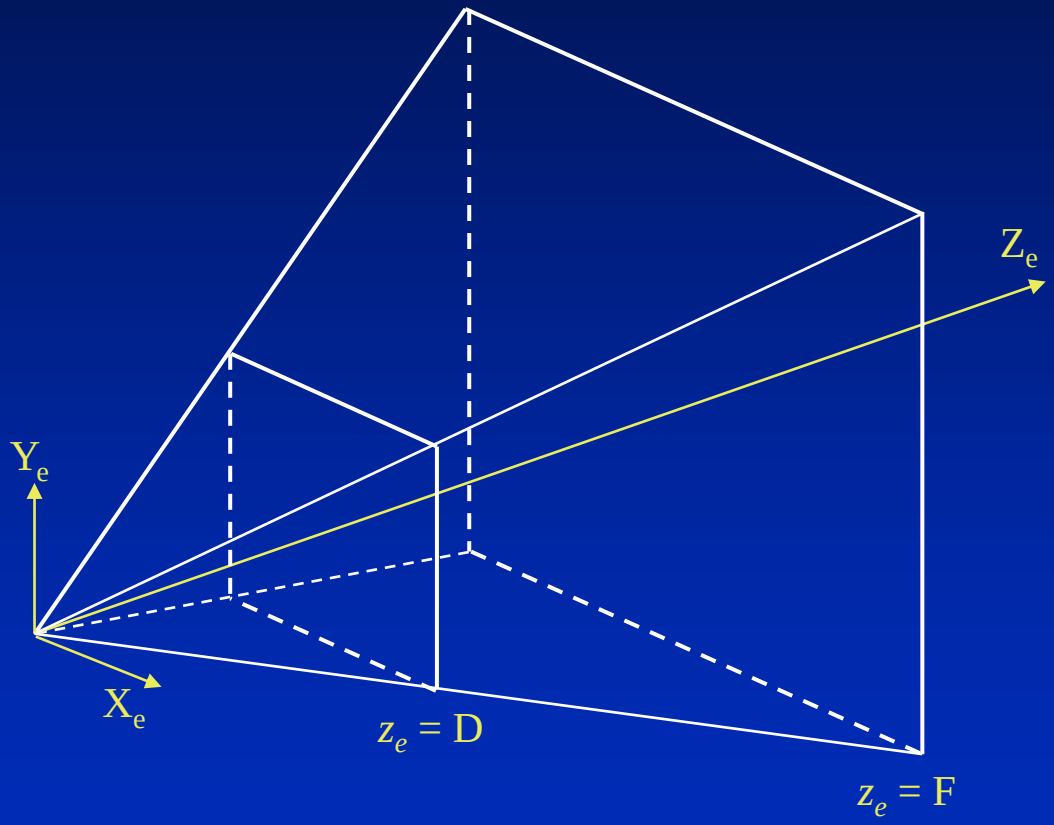


http://upload.wikimedia.org/wikipedia/en/5/5f/DoF-sym.png

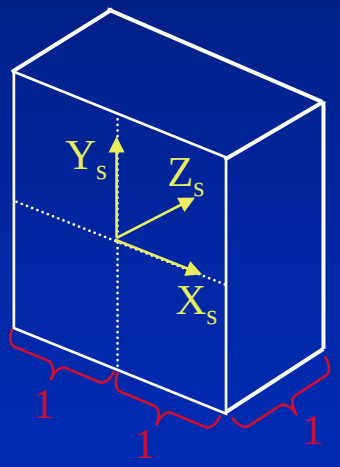
Perspective Projections in Computer Graphics



Mapping a Viewing Frustum to a Standard Viewbox



Frustum of vision



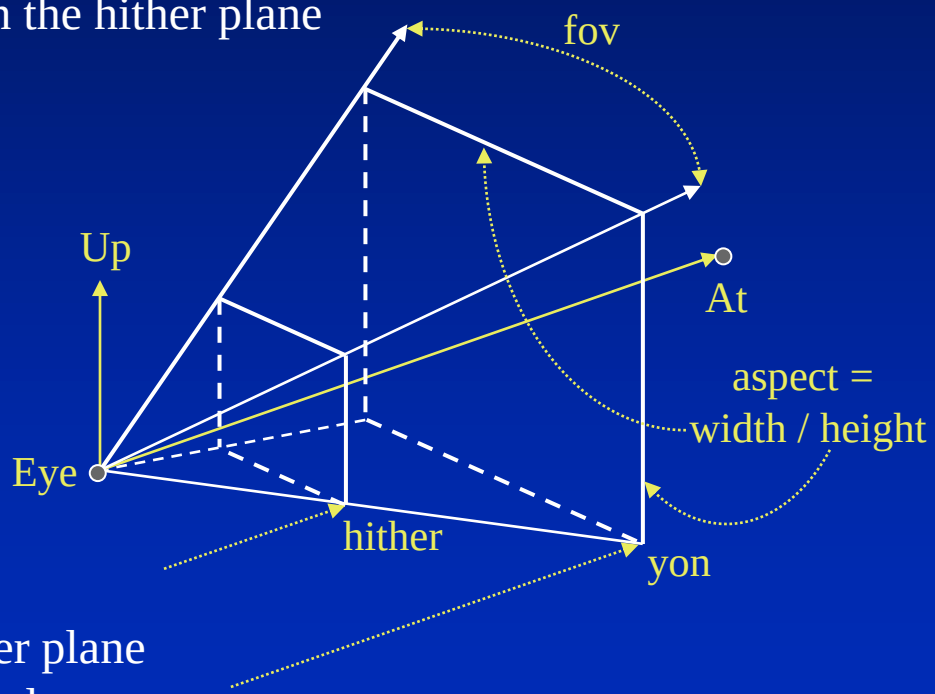
Screen coordinate system

Cornell Program of

Viewing Frustum - User Parameters

Location and Orientation (vectors):

Eye	location of eye point
At	a point along view direction vector
Up	a vector defining the Y axis on the hither plane



Shape (scalars):

fov	field of view angle
aspect	view plane aspect ratio
hither	perpendicular distance to hither plane
yon	perpendicular distance to yon plane

Object Space

(One point perspective)

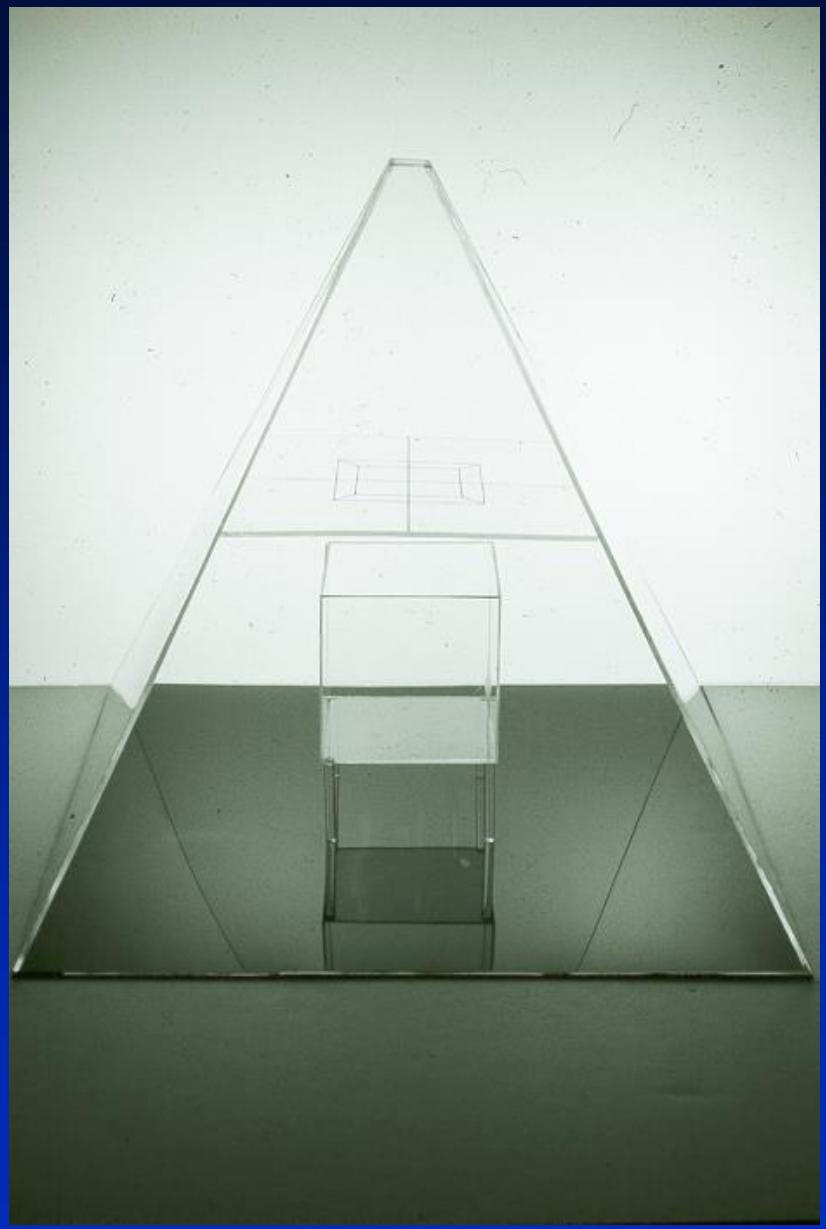
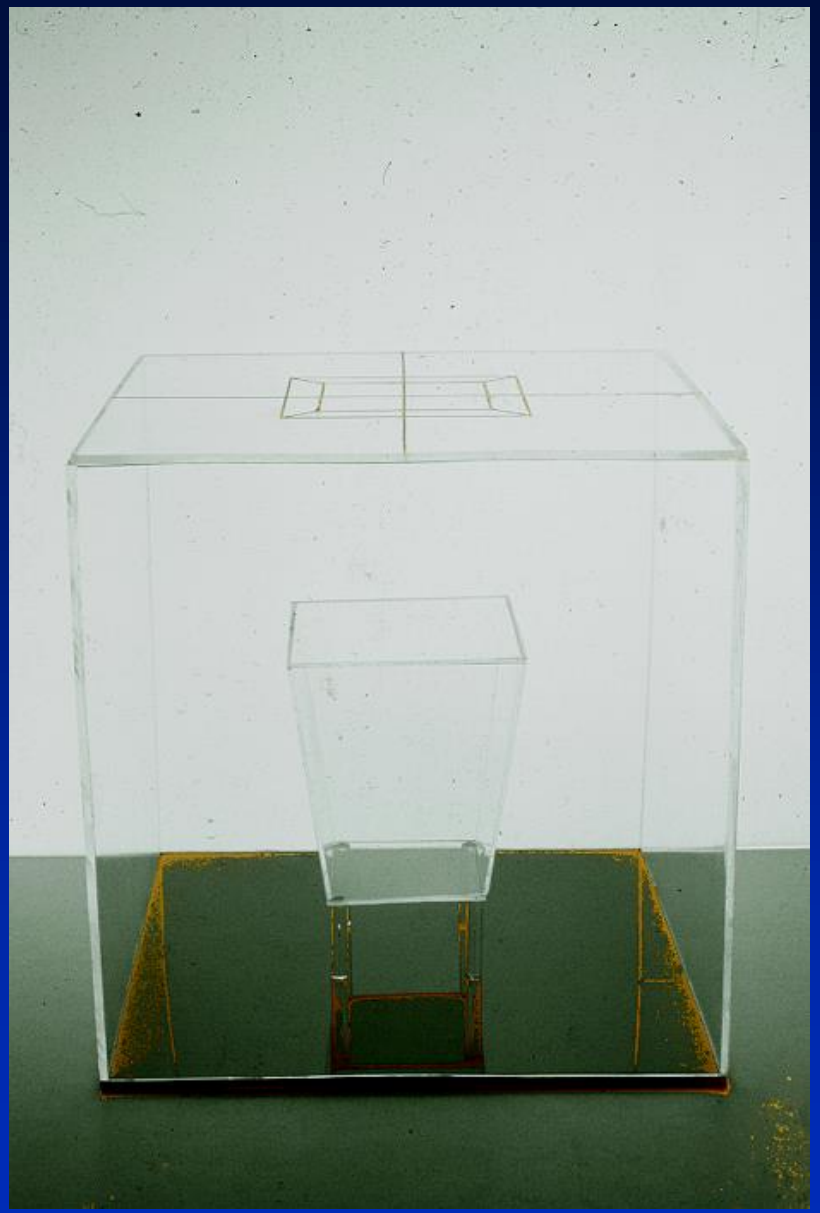


Image Space

(One point perspective)



Object Space

(Two point perspective)

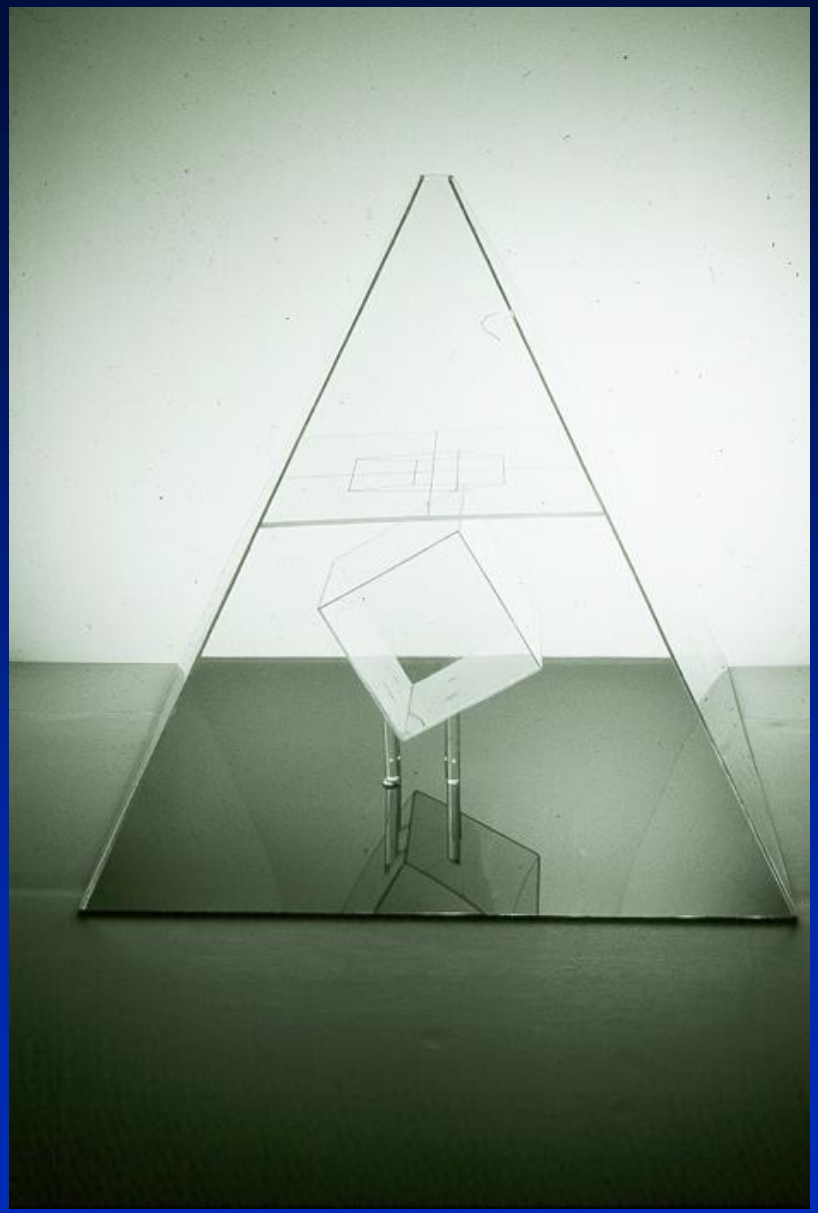
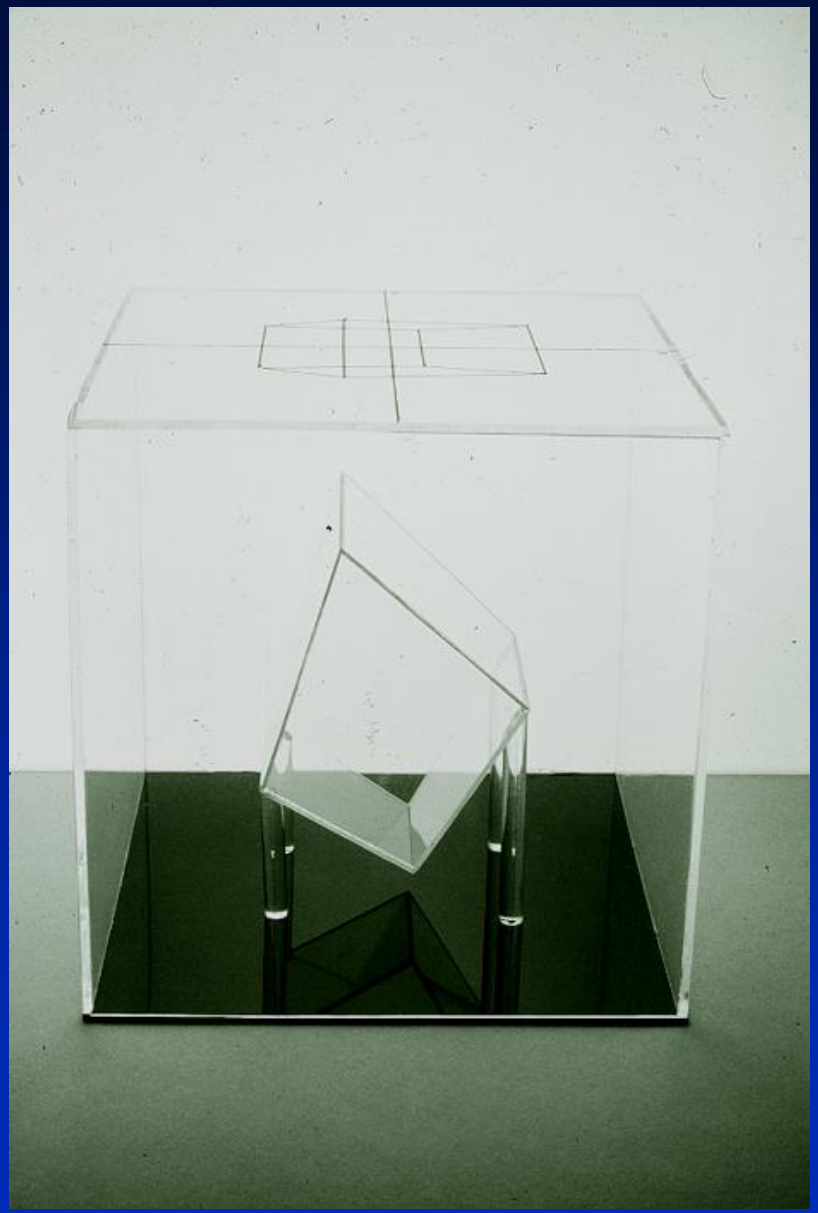


Image Space

(Two point perspective)



Object Space

(Three point perspective)

Image Space

(Three point perspective)

3D Perspective Image (x & y coordinates in virtual image space)

$$x_p = -f\frac{x_e}{z_e} \qquad y_p = -f\frac{y_e}{z_e}$$

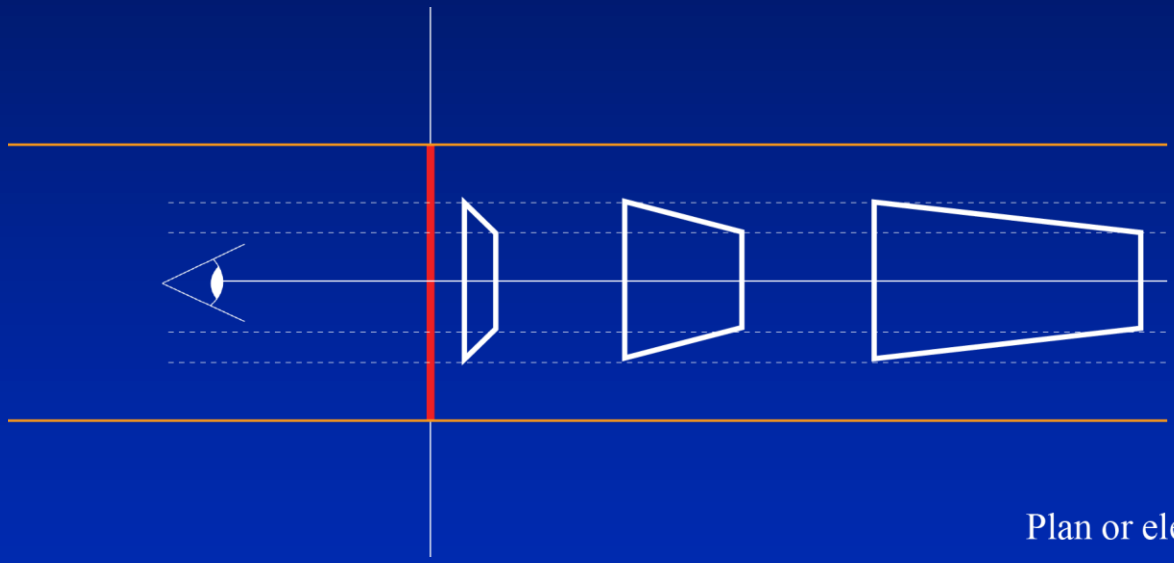
$$x_p = \text{constant}\frac{x_e}{z_e} \qquad y_p = \text{constant}\frac{y_e}{z_e}$$

3D Perspective Image (z depth coordinate in virtual image space)

$$z_p = -2f - \frac{f^2}{z_e}$$

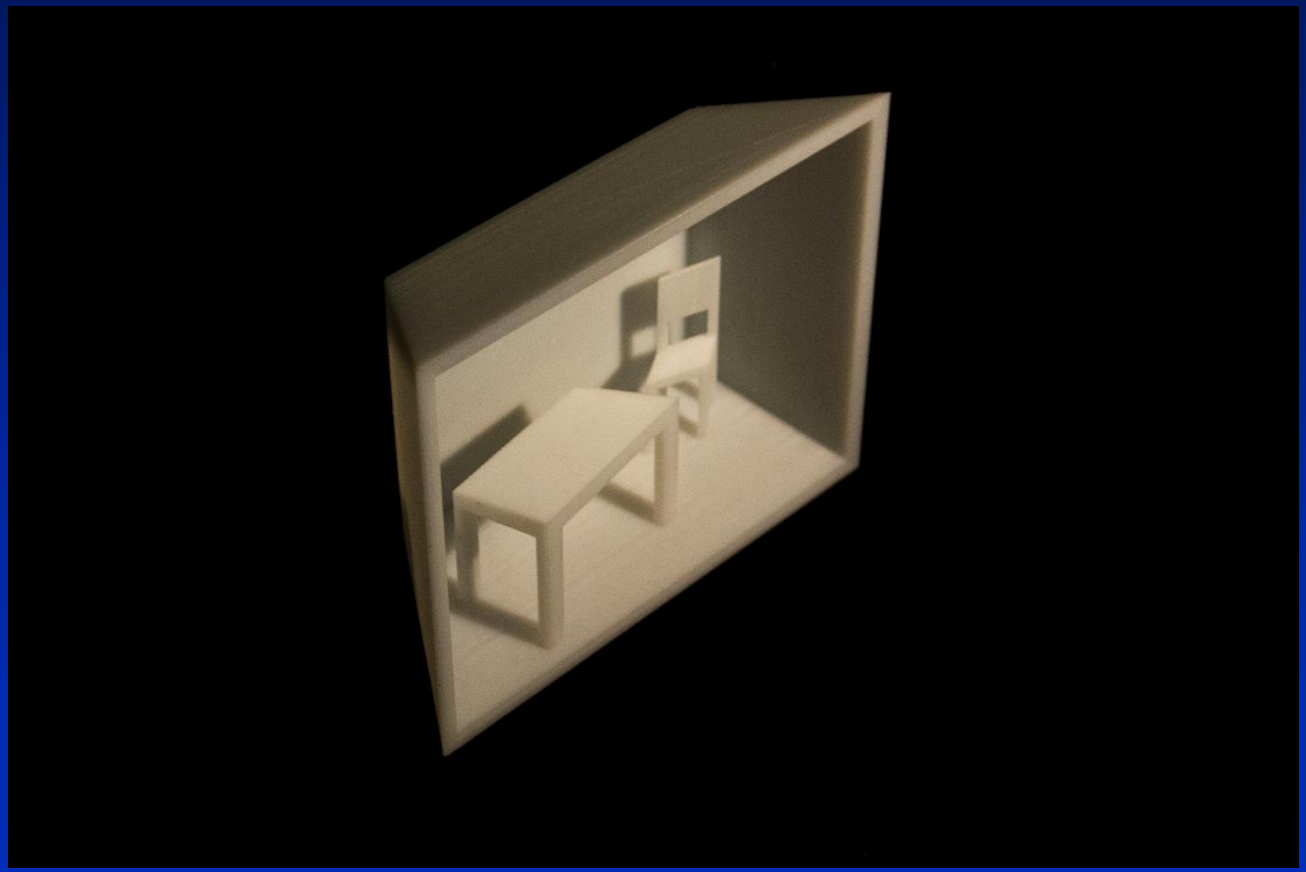
$$z_p = \text{constant}_1 + \text{constant}_2 \frac{1}{z_e}$$

Equivalent Distorted Geometries in Virtual Image Space

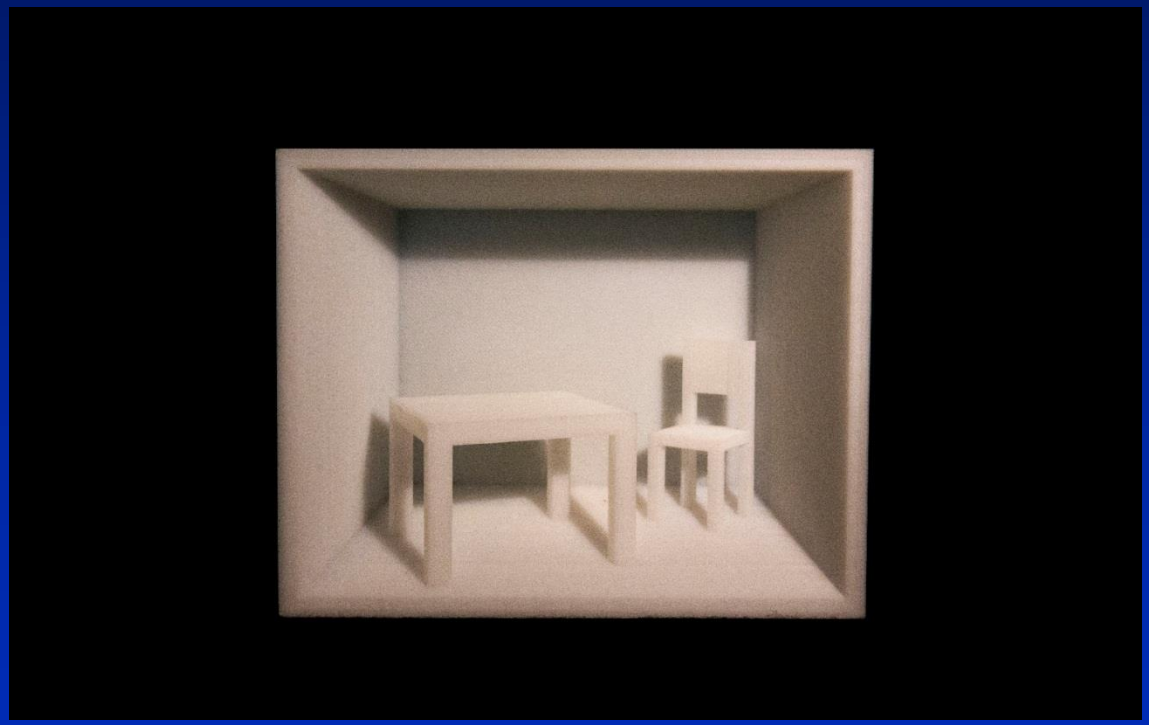


Plan or elevation view

Distorted Geometry Model



Distorted Geometry Model



Uses of Perspective in Enlarging Architectural Space (3D)

Palazzo Spada

Borromini

Palazzo Spada

Borromini

Imss,Firenze 2004 - 3 Scenografia prospettica

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Palazzo Spada

Borromini

Empire of the Eye” The magic illusion Palazzo Spada Borromini

End. . .