Single Scattering in Refractive Media with Triangle Mesh Boundaries

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Single Scattering

- Direct illumination in refractive objects is hard

Single scatter from a single point light source
Single Scattering

- Direct illumination in refractive objects is hard

Single scatter from a single point light source
Single Scattering

- Direct illumination in refractive objects is hard

Single scatter from a single point light source
Problem

Eye ray
Problem

• Find direct illumination at V (receiver) from L (light)
Problem

- Find direct illumination at V (receiver) from L (light)

Ignoring refraction
Challenges: Bending of Path

- Find direct illumination at V (receiver) from L (light)
- Light bends at interface according to Snell’s Law

Ignoring refraction

With refraction

Eye ray
Challenges: Multiple Paths

- Find direct illumination at V (receiver) from L (light)
- Light bends at interface according to Snell’s Law
  - Can have zero, one, or many such paths (and P’s)

Ignoring refraction

With refraction

Eye ray

V

P

L

P₁

P₂
Is it important?

- Glass tile quality comparison

Shadow rays ignore refraction

Our method
Challenges Summary

• Bending of paths
• Multiple paths
• Shading normals
• Large triangle meshes
Challenges: Shading Normals

- Geometric normals ($N_G$) vs. shading normals ($N_s$)
  - E.g., interpolated normals, bump maps, normal maps
  - Alters directions and intensities of light paths
Challenges: Shading Normals

- Geometric normals ($N_G$) vs. shading normals ($N_S$)
  - E.g., interpolated normals, bump maps, normal maps
  - Alters directions and intensities of light paths
Limitations

- Finds connections that
  - Cross the boundary exactly once
  - Have no other changes in direction
  - Cost depends on path count and boundary
Prior Work

• Diffusion and multiple scatter
  – [eg, Stam 95, Jensen et al. 01, Wang et al. 08]

• Monte Carlo
  – [eg, Kajiya 86, Veach 97]

• Beam tracing
  – [eg, Nishita & Nakamae 94, Iwasaki et al. 03, Ernst et al. 05]

• Photon mapping
  – [eg, Jensen 01, Sun et al. 08, Jarosz et al. 08]

• Fermat’s principle
  – [eg, Mitchell & Hanrahan 92, Chen & Arvo 00]
Prior Work

• Mitchell & Hanrahan 92
  – Used Fermat’s principle and Newton’s method
  – Reflection (shown) and refraction

• Limitations
  – Only supported implicit surfaces
  – Cannot handle shading normals
  – Expensive 3D search
  – Not scalable to complex geometry
Contributions

• Support triangles with shading normals
  – Most widely used geometry format
  – Required fundamental problem reformulation
  – New search methods and intensity equations

• Hierarchical culling
  – Scales to complex objects with many triangles

• CPU and GPU implementations
  – Interactive performance on some scenes
Outline

• Half-vector formulation
• Solving for a single triangle
• Hierarchical culling for meshes
• Results
Fermat’s Principle

- Define optical path length
  - $d(P) = \eta ||V-P|| + ||P-L||$
  - Extrema of $d(P)$ are the refraction points
- Cannot handle shading normals

Index of refraction = $\eta$
• Used in micro-facet model [Walter et al. 07]

• Direction to receiver: $\omega_V = (V - P) / ||V - P||$

• Direction to light: $\omega_L = (L - P) / ||L - P||$
Half-Vector Formulation

- Used in micro-facet model [Walter et al. 07]
- Direction to receiver: $\omega_V = (V - P) / \|V - P\|$
- Direction to light: $\omega_L = (L - P) / \|L - P\|$
- Half-vector: $H = (\eta \omega_V + \omega_L) / \|\eta \omega_V + \omega_L\|$
Half-Vector Formulation

• If $H = -N$ (surface normal) then
  – $V$, $P$, $L$, and $N$ are coplanar
  – Angles obey Snell’s Law: $\eta \sin(\theta_V) = \sin(\theta_L)$

• It is a refraction solution
  – Assuming $V$ and $L$ lie on the correct sides of the normal
Half-Vector Formulation

• Find all $P$ such that: $H + N = 0$
  
  – Natural extension to shading normals: $H + N_s = 0$

• Newton’s method to find zeroes of: $f(P) = H + N$
Newton’s Method Review

- Quadratically convergent near a root
  - Each iteration doubles the precision
- Chaotic behavior far from a root
  - May diverge or converge to other roots
Outline

• Half-vector formulation
• Solving for a single triangle
  – Geometric normal - 1D Newton
  – Shading normals - 2D Newton
  – Subdivision oracles
• Hierarchical culling for meshes
• Results
Solution must lie in plane containing $V$, $L$, and $N_G$:

- Unique solution always exists
- Simple 1D Newton’s method converges
  - Typically in just 2 to 4 iterations
- Check if solution lies within the triangle
• Solution must lie in plane containing V, L, and NG
  – Unique solution always exists
  – Simple 1D Newton’s method converges
    • Typically in just 2 to 4 iterations
  – Check if solution lies within the triangle
Triangle with Shading Normal

- Shading normal, $N_S$, varies over triangle
  - Full 2D search over triangle’s area
- Function $f(P) = H + N_S$ maps 2D to 3D
  - Derivative is 2X3 Jacobian matrix is non-invertible
  - Use pseudoinverse for Newton’s method:

$$J^+ = (J^T J)^{-1} J^T$$

$$X_{i+1} = X_i - J^+(X_i)f(X_i)$$
Triangle with Shading Normal

• Need good starting points
• May have zero, one, or multiple solutions
  – Subdivide triangle as needed to isolate solutions
Two Triangle Subdivision Oracles

• Test with strong guarantees
  – Based on [Krawczyk 69], [Mitchell&Hanrahan 92]
  – Conditions guarantee uniqueness and convergence

• Fast empirical heuristic
  – Based on solid angles of triangle and normals
Triangle summary

• For each triangle:
  – If no shading normals
    • Solve for P with 1D Newton
  – Else if passes the subdivision oracle
    • Solve for P with 2D Newton
  – Else
    • Subdivide into 4 triangles and try again
  – Test if P lies within the triangle
Outline

• Half-vector formulation
• Solving for a single triangle
• Hierarchical culling for meshes
• Results
Culling Tests

- Most triangles contain no solutions for P
- Three quick culling tests
  - Spindle
  - Sidedness
  - Interval
Spindle Culling Test

- Refraction bends path by angle $\leq \arccos(1/\eta)$
  - Solutions must lie within circular arc (2d) or spindle (3d)
Spindle Culling Test

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Spindle Culling Test

- Refraction bends path by angle $\leq \arccos\left(\frac{1}{\eta}\right)$
  - Solutions must lie within circular arc (2d) or spindle (3d)
Sidedness Culling Test

- Light L must be on the outside of surface at P
- Receiver V must be inside within the critical angle

\[ \arccos\left(\frac{1}{n}\right) \]
Hierarchical Culling for Meshes

• Apply culling test to groups of triangles
• Use 6D position-normal tree [Bala et al. 03]
  – Leaves are boundary triangles
  – Boxes and cones bound positions and normals
  – Traverse top-down
Algorithm Summary

• Build position-normal tree for each boundary mesh
• For each eye ray
  – Trace until hits surface or volume-scatters at V
  – Select a light source point, L
  – Traverse tree to solve for all P on boundary
  – For each solution point P
    • Check for occlusion along path
    • Compute effective light distance
    • Add contribution to pixel value
Effective Distance to Source

• Refraction alters usual $1/r^2$ intensity falloff
  – Can focus or defocus the light

• Compute effective light distance for each path
  – Simple formula for triangles without shading normal
  – Use ray differentials [Igehy 99] for shading normal case
  – See paper for details
Outline

• Half-vector formulation
• Solving for a single triangle
• Hierarchical culling for meshes
• Results
Results - CPU

• Three scenes without shading normals

Teapot  Cuboctahedron  Amber
Results - CPU

- Three scenes without shading normals

Teapot
15.3s

Cuboctahedron
13.9s

Amber
19.2s

512x512 images, 64 samples per pixel, 8-core 2.83GHz Intel Core2
Results - Teapot

- Teapot quality comparison

Shadow rays ignore refraction

Our method (15.4s)
Results - Teapot

- Teapot quality comparison

Shadow rays ignore refraction  
Our method (15.4s)
Results - Cuboctahedron

- Cuboctahedron movie (13.9s)
Results - GPU

- Implemented on GPU using CUDA 2.0
- 1D, 2D Newton iteration
- Hierarchical pruning
- Ray tracing based on [Popov et al. 2007]
- One kernel thread per eye ray
- Does not yet support all scenes
## Results - GPU

<table>
<thead>
<tr>
<th>Name</th>
<th>Render Time</th>
<th>FPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teapot</td>
<td>0.1 s</td>
<td>10 fps</td>
</tr>
<tr>
<td>Cuboctahedron</td>
<td>0.14 s</td>
<td>7 fps</td>
</tr>
<tr>
<td>Amber</td>
<td>0.3 s</td>
<td>3 fps</td>
</tr>
</tbody>
</table>

512x512 images, 2 eye rays per pixel + 40-60 volume samples, nVIDIA GTX 280, CUDA 2.0
Results - GPU

- Teapot example
  - 10 fps on GPU
Results - CPU

• Three scenes with shading normals

Glass tile
Glass mosaic
Pool
Results - CPU

• Three scenes with shading normals

Glass tile 66.9s
Glass mosaic 87.8s
Pool 59.4s

512x512 images, 64 samples per pixel, 8-core 2.83GHz Intel Core2
Results - Glass Tile

Photon map (equal time)  Our method (66.9s)
Results - Glass Mosaic

- Glass mosaic movie (87.8s)
Results - Component Evaluation

• Evaluation of algorithm components
  – Pool (2632 triangles in boundary)

<table>
<thead>
<tr>
<th></th>
<th>Time</th>
<th>Ratio</th>
</tr>
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<tbody>
<tr>
<td>Without Hierarchy</td>
<td>1934.6s</td>
<td>32x</td>
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<tr>
<td>Guaranteed Convergence</td>
<td>141.1s</td>
<td>2.4x</td>
</tr>
<tr>
<td>Subdivision Heuristic</td>
<td>59.4s</td>
<td>1x</td>
</tr>
</tbody>
</table>
Results - Bumpy sphere

- Bumpy sphere (9680 triangles)
  - Volume sampling noise
    - Used 128 samples per pixel
    - Effective distance clamping

Our method 304.3 s
Results - Bumpy sphere

- Bumpy sphere (9680 triangles)

Shadow rays ignore refraction

Our method
Results - Bumpy sphere

• Bumpy sphere (9680 triangles)

Path tracing (16x time)  Our method
Results - Bumpy sphere

- Bumpy sphere (9680 triangles)

Photon map 10M (equal time)  Our method
Conclusion

- New method for single scatter in refractive media
  - Applicable to many rendering algorithms
  - New half-vector formulation
  - Efficient culling and search methods
  - Supports shading normals and large triangle meshes
  - Interactive performance for some scenes

- Future work
  - Better culling
  - Reflections and low-order scattering
  - Multiple interfaces
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  – Intel
  – NVidia
  – Microsoft
  – INRIA sabbatical program

• PCG Graphics Lab and Elizabeth Popolo
The End
## Results - CPU timings

<table>
<thead>
<tr>
<th>Name</th>
<th>Render Time</th>
<th>Triangles</th>
<th>Shading Normals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Surface</td>
<td>Other</td>
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<tr>
<td>Teapot</td>
<td>15.3 s</td>
<td>12</td>
<td>4096</td>
</tr>
<tr>
<td>Cuboctahedron</td>
<td>13.9 s</td>
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<tr>
<td>Amber</td>
<td>19.2 s</td>
<td>36</td>
<td>60556</td>
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<tr>
<td>Glass tile</td>
<td>66.9 s</td>
<td>798</td>
<td>60</td>
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<tr>
<td>Glass mosaic</td>
<td>87.8 s</td>
<td>20813</td>
<td>1450</td>
</tr>
<tr>
<td>Pool</td>
<td>59.4 s</td>
<td>2632</td>
<td>4324</td>
</tr>
<tr>
<td>Bumpy Sphere</td>
<td>304.3 s</td>
<td>9680</td>
<td>0</td>
</tr>
</tbody>
</table>

512x512 images, 64 samples per pixel (128 for bumpy sphere), 8-core 2.83GHz Intel Core2 CPU
Newton’s Method

• Iterative root finding method
  – Start with initial guess \( x_0 \)
  – Iteration: \( x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \)
Newton’s Method

• Iterative root finding method
  – Start with initial guess x₀
  – \( x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \)
Newton’s Method

• Iterative root finding method
  – Start with initial guess $x_0$
  – $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$