Visual Imaging and the Electronic Age

Color Science

Metamers & Chromaticity Diagrams

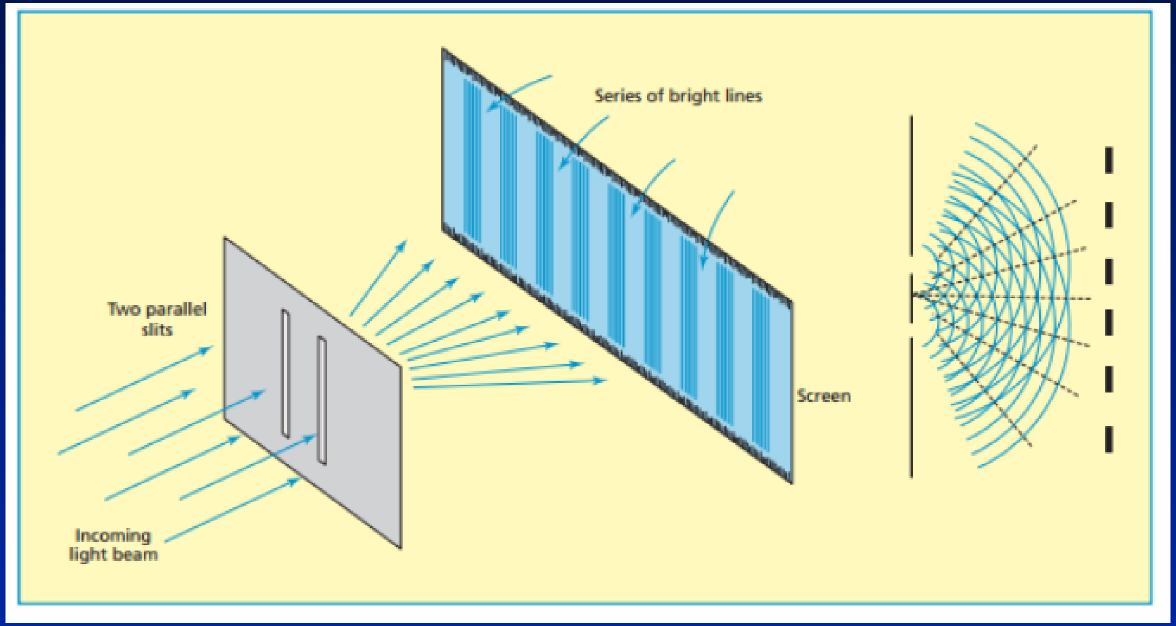
Lecture #6

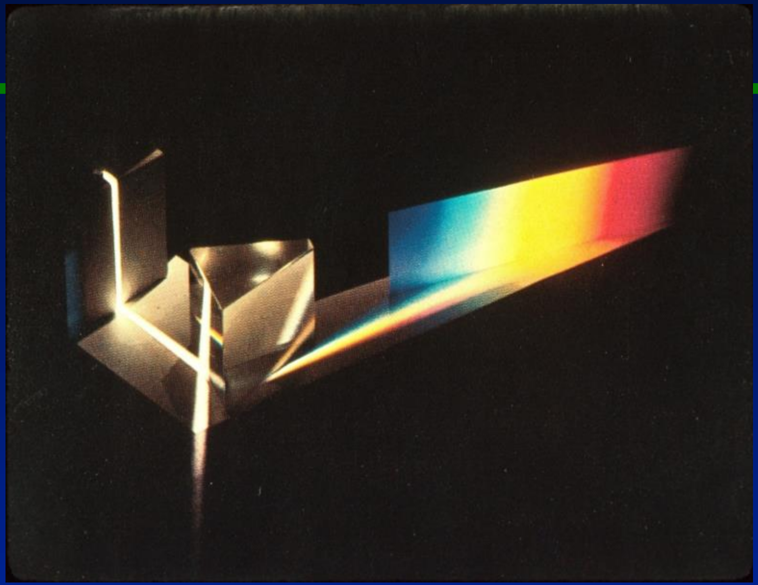
September 22, 2020

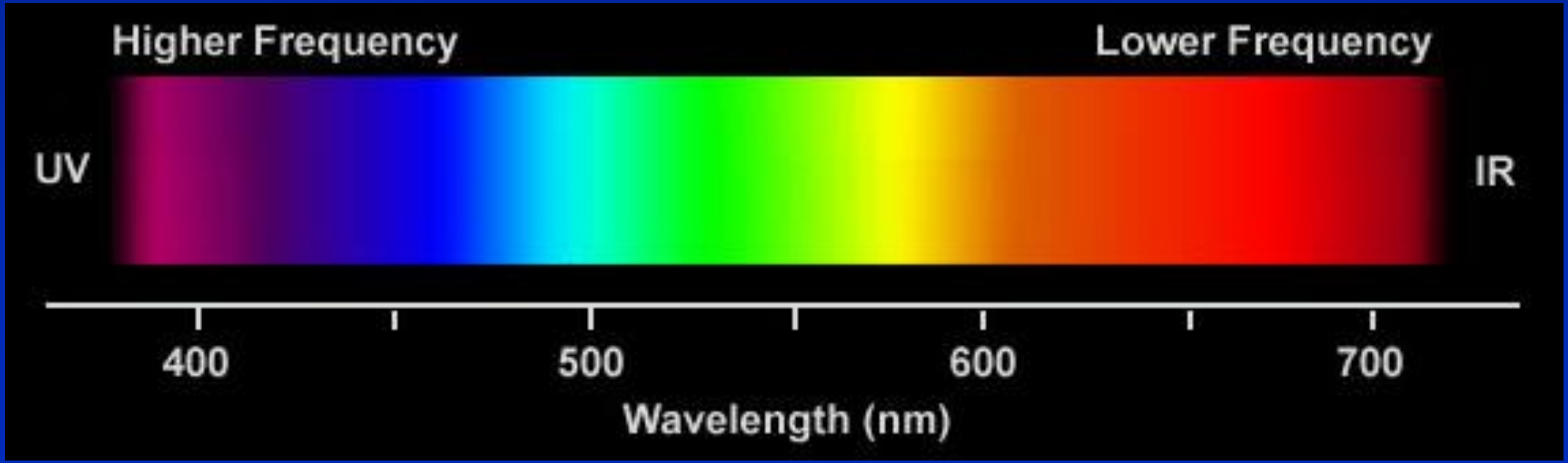
Prof. Donald P. Greenberg

Review

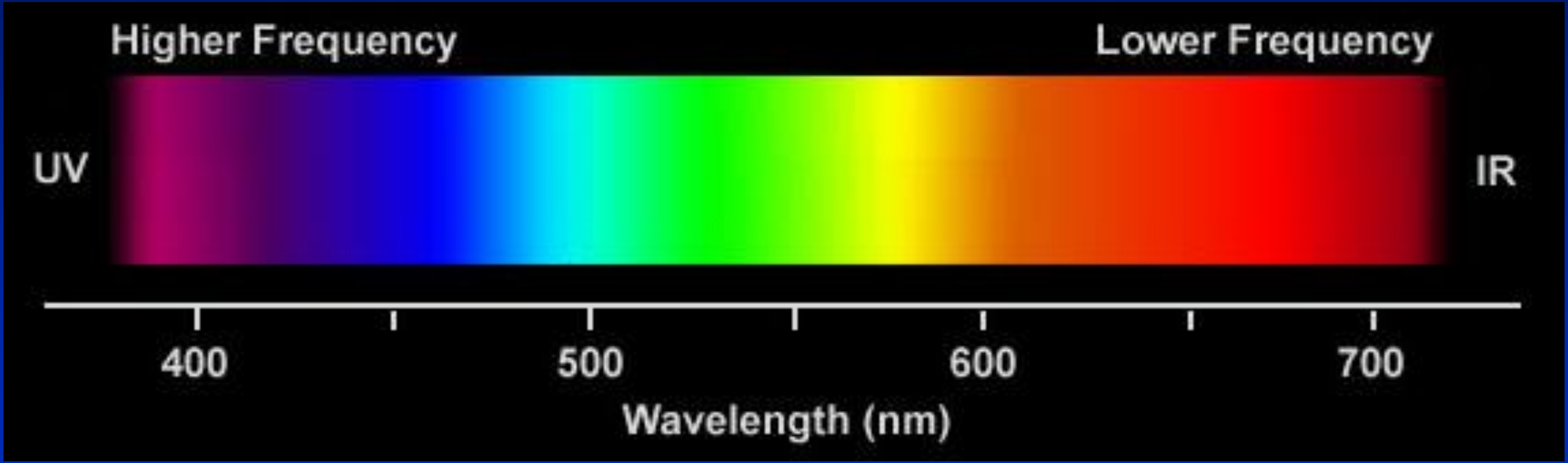
Light as Waves





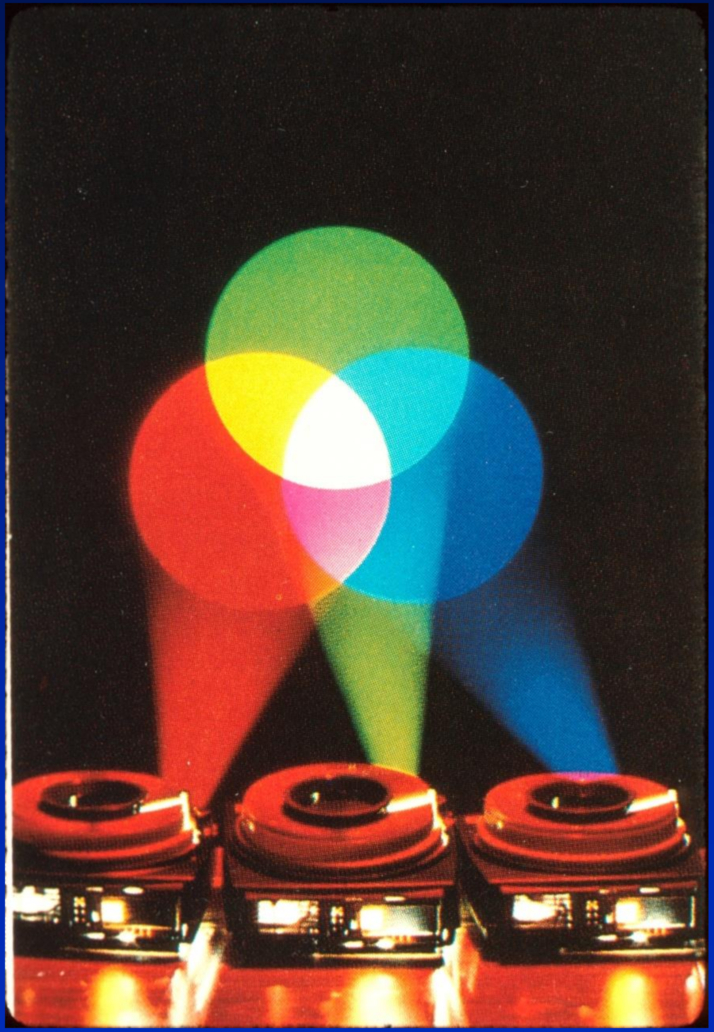


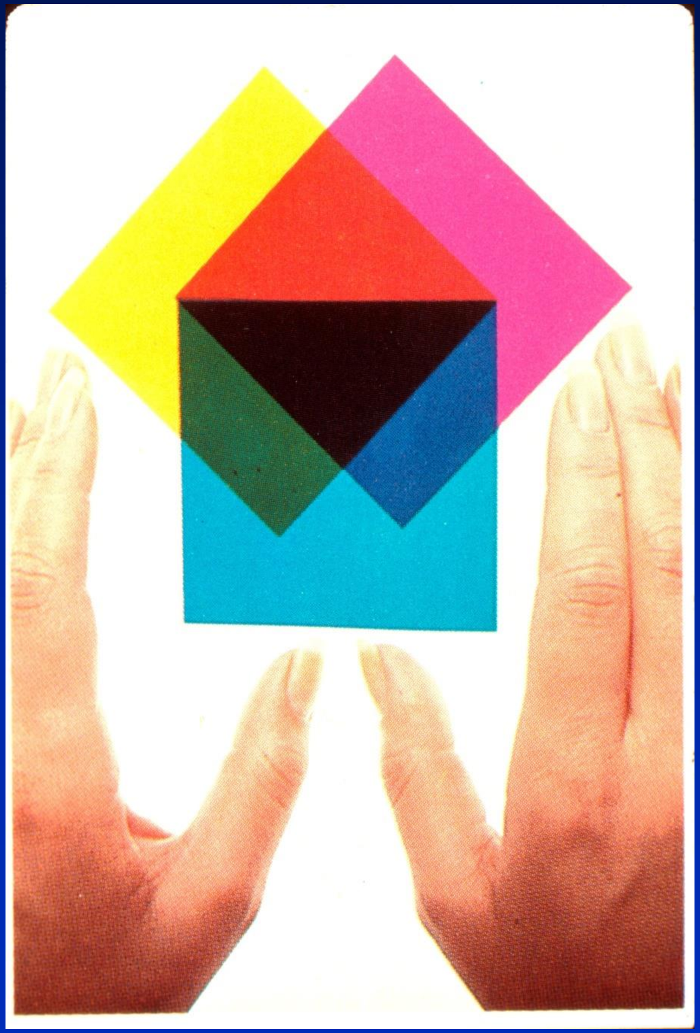
Visible Light Spectrum



Additive & Subtractive Color Spaces

RGB





CMY

Emitted, Reflected, and Transmitted Light

$$P = \int_{390}^{700} E_\lambda \cdot d_\lambda$$

$$P = \int_{390}^{700} E_\lambda \cdot p_\lambda \cdot d_\lambda$$

$$P = \int_{390}^{700} E_\lambda \cdot t_\lambda \cdot d_\lambda$$

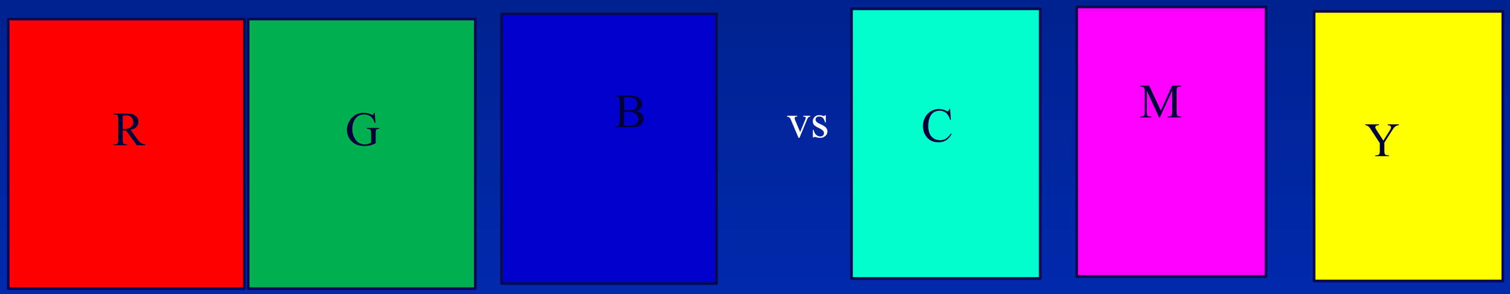
End of review

Question?

How can we reproduce such a vast range of color using two completely different sets of three primary colors?

Emissive (Displays)

Reflective (Printers)



Additive Technology

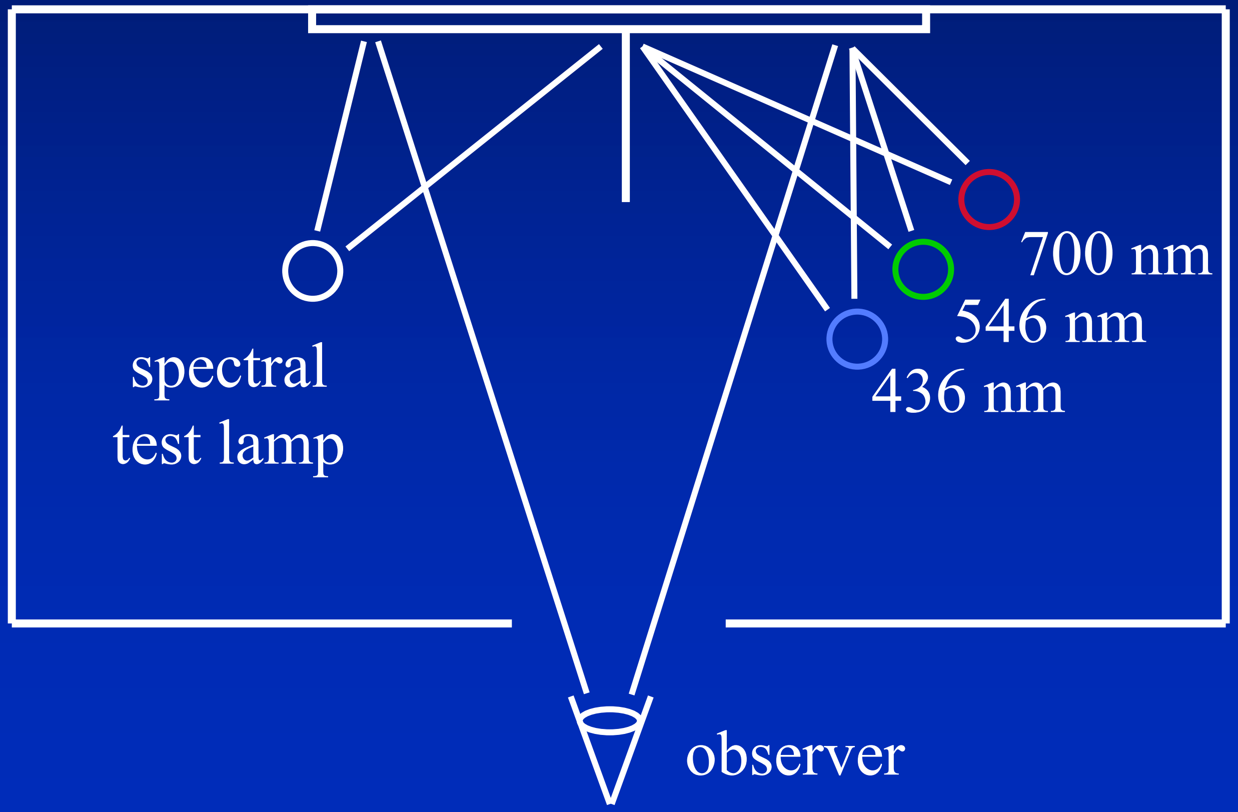
Subtrative Technology

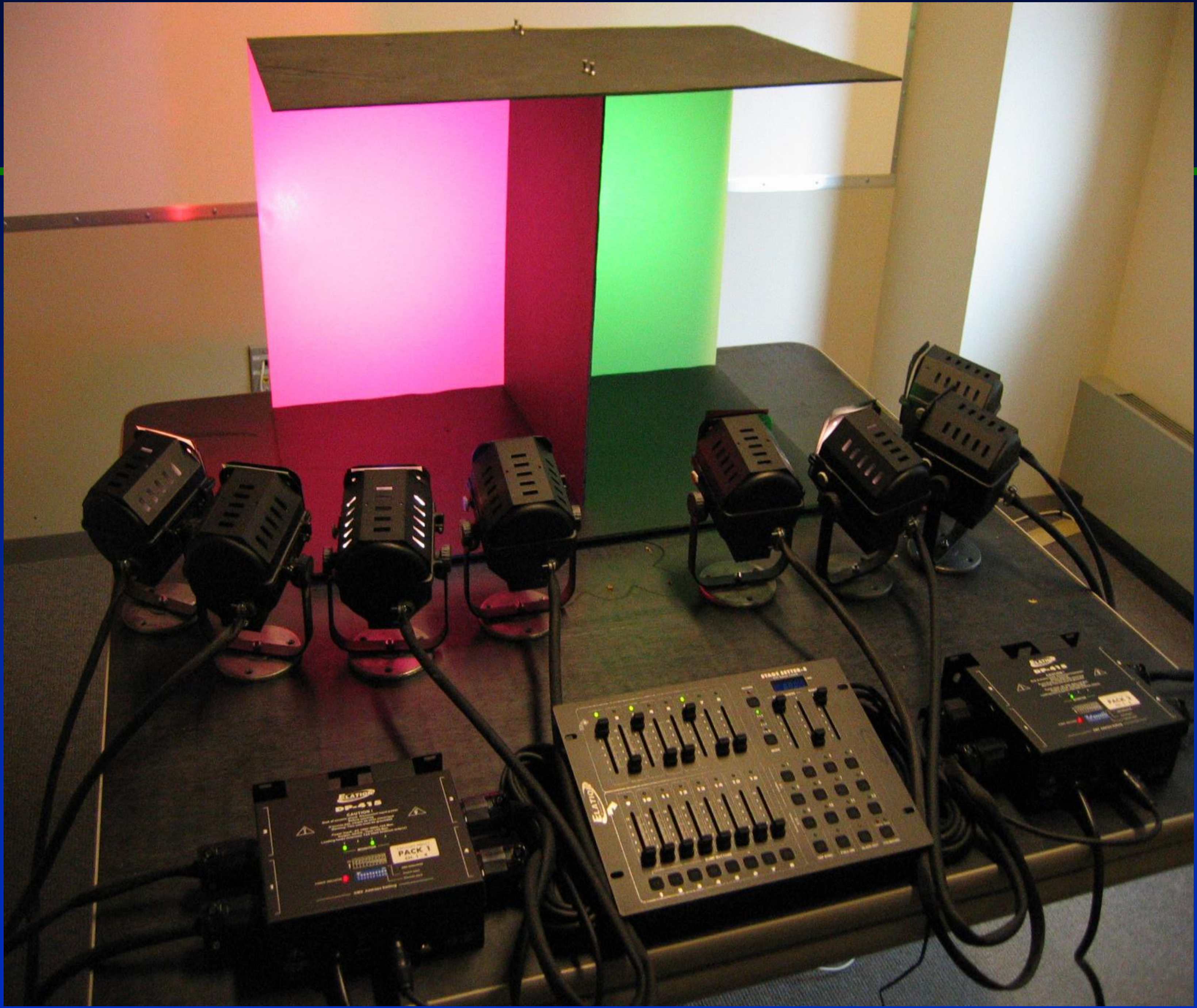
Grassmann's Color Matching Experiments (1853)

- Roy. S. Berns. Billmeyer and Saltzman's PRINCIPLES OF COLOR TECHNOLOGY, 2000 John Wiley & Sons, Inc.

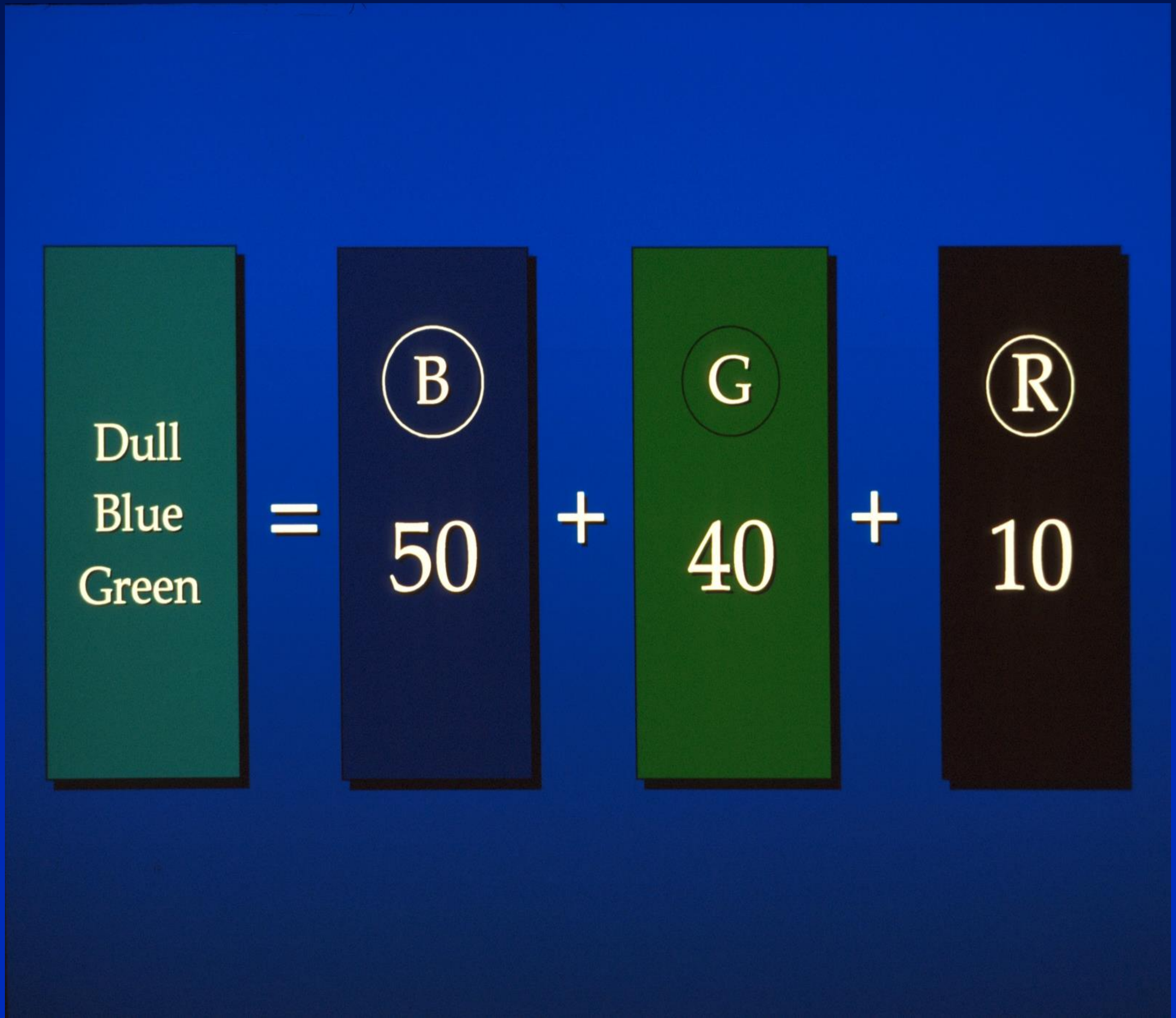
Matching a Test Lamp with 3 Primary Lights

- We can match a color sensation from *any* spectrum using only 3 primary colors (R,G,B)

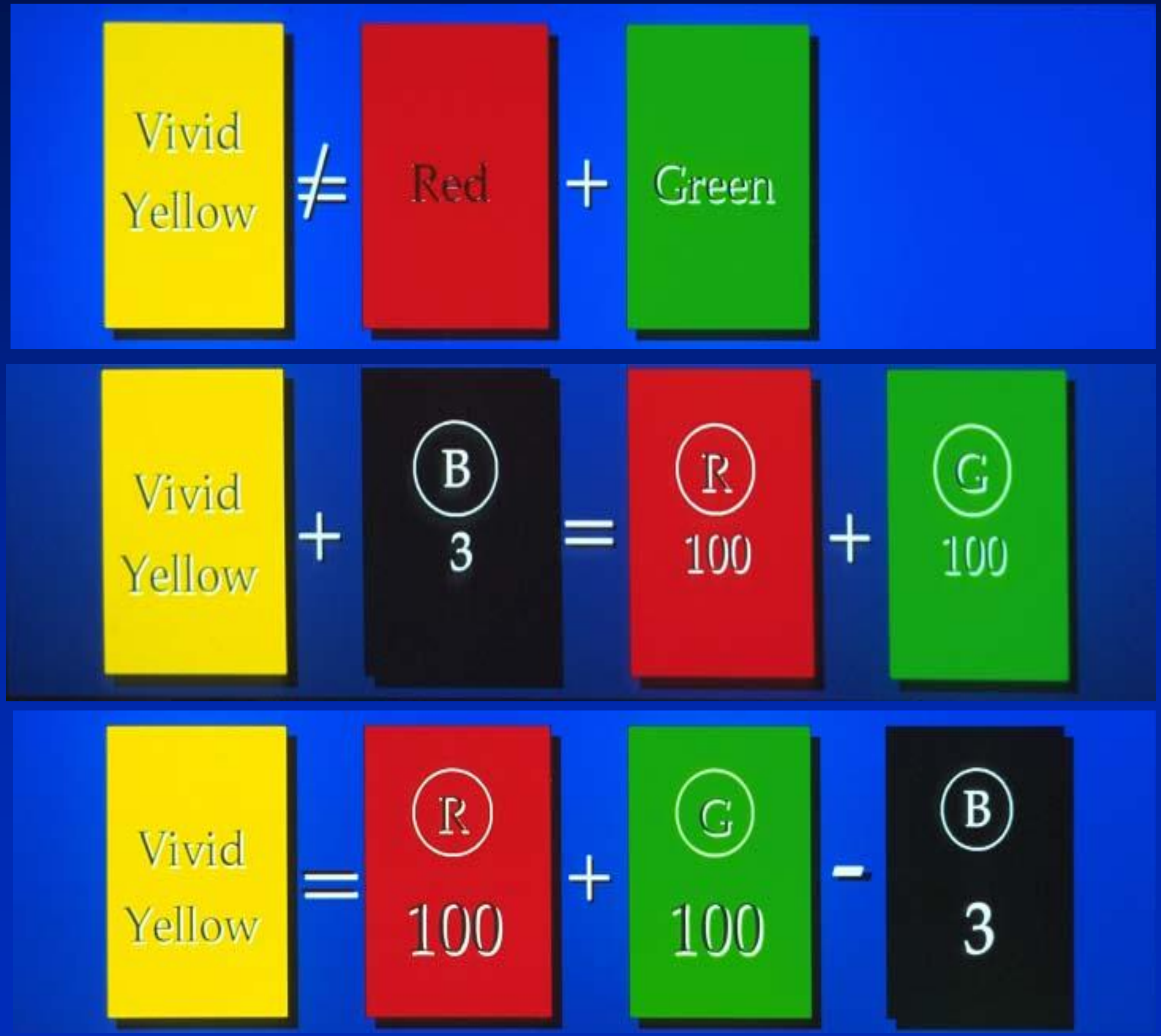



ELATION
CAUTION !
PACK 1

Matching a Test Lamp with 3 Primary Colors

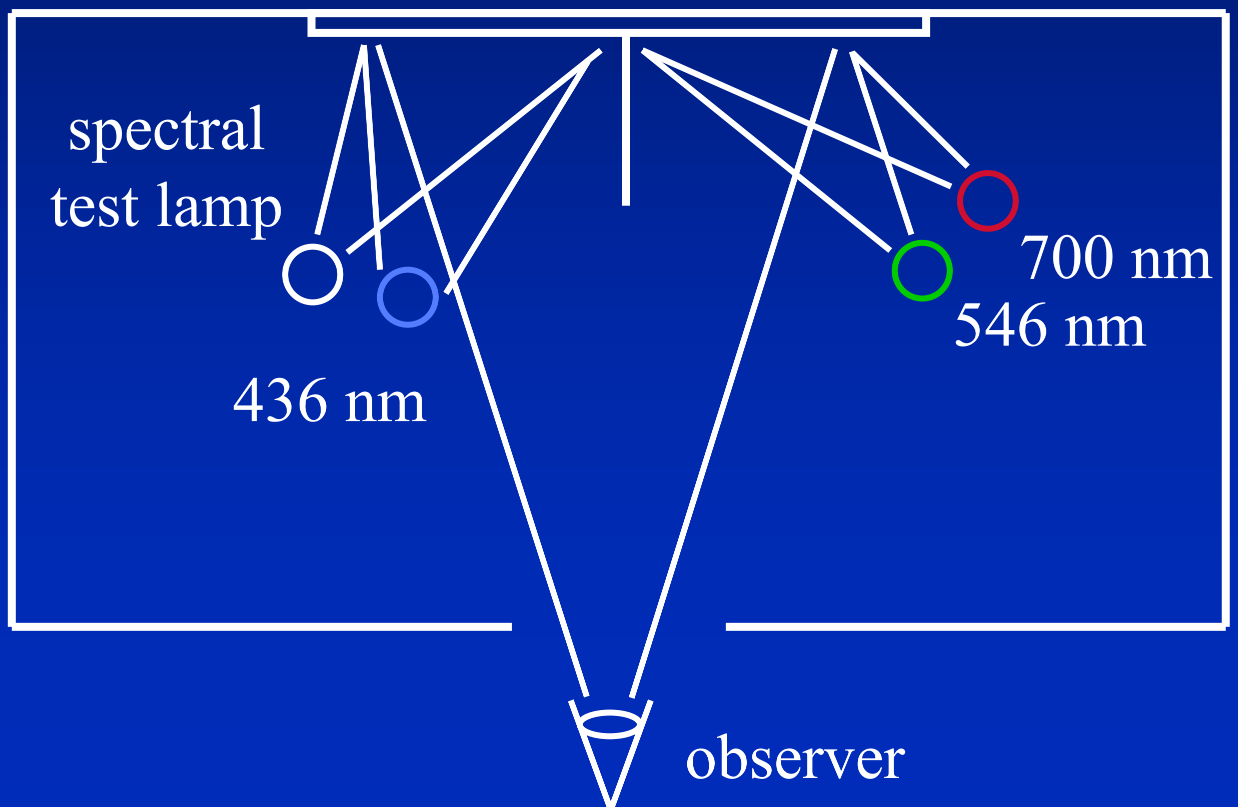


Matching a Test Color (Lamp) with 3 Primary Colors

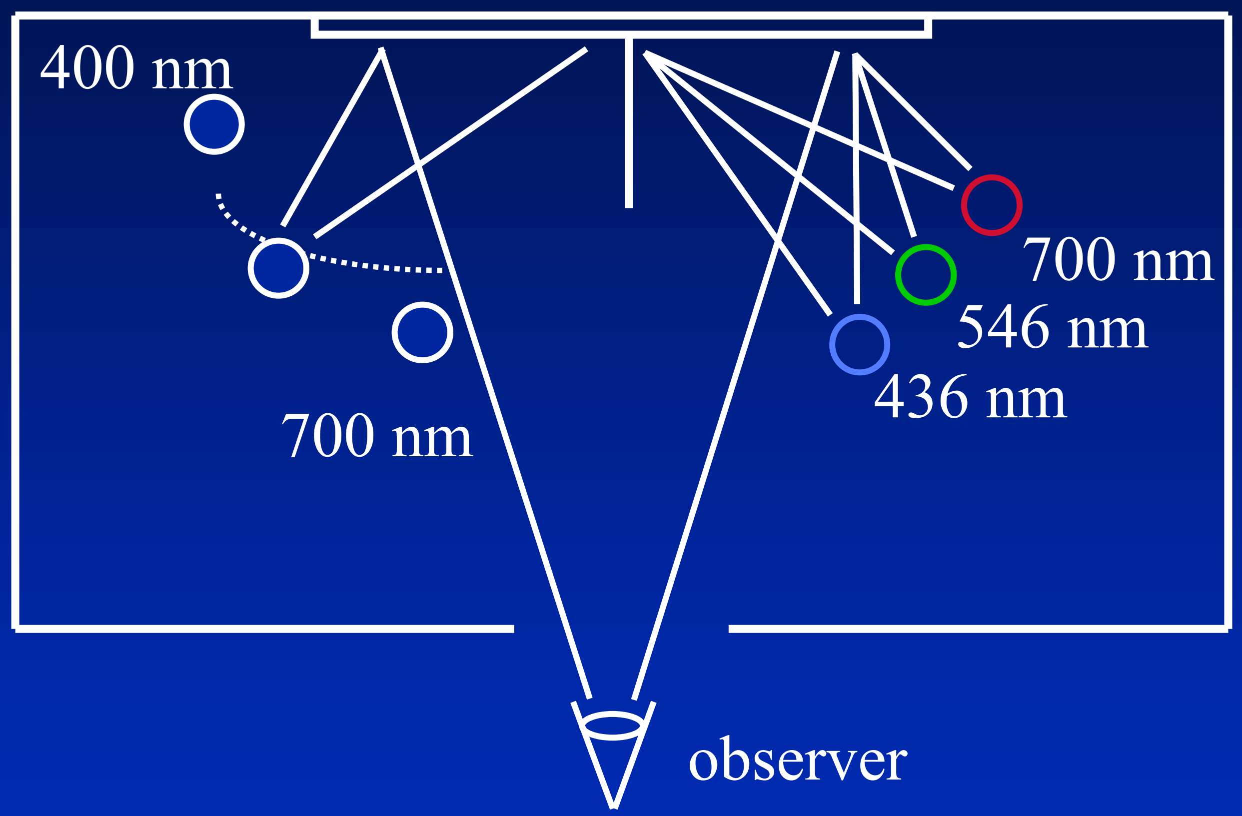


Matching a Test Lamp with 3 Primary Lights

- Need to allow “negative light”
  - Can’t match a bright yellow (Y) light with R,G,B.
  - But can match Y + B with R + G.

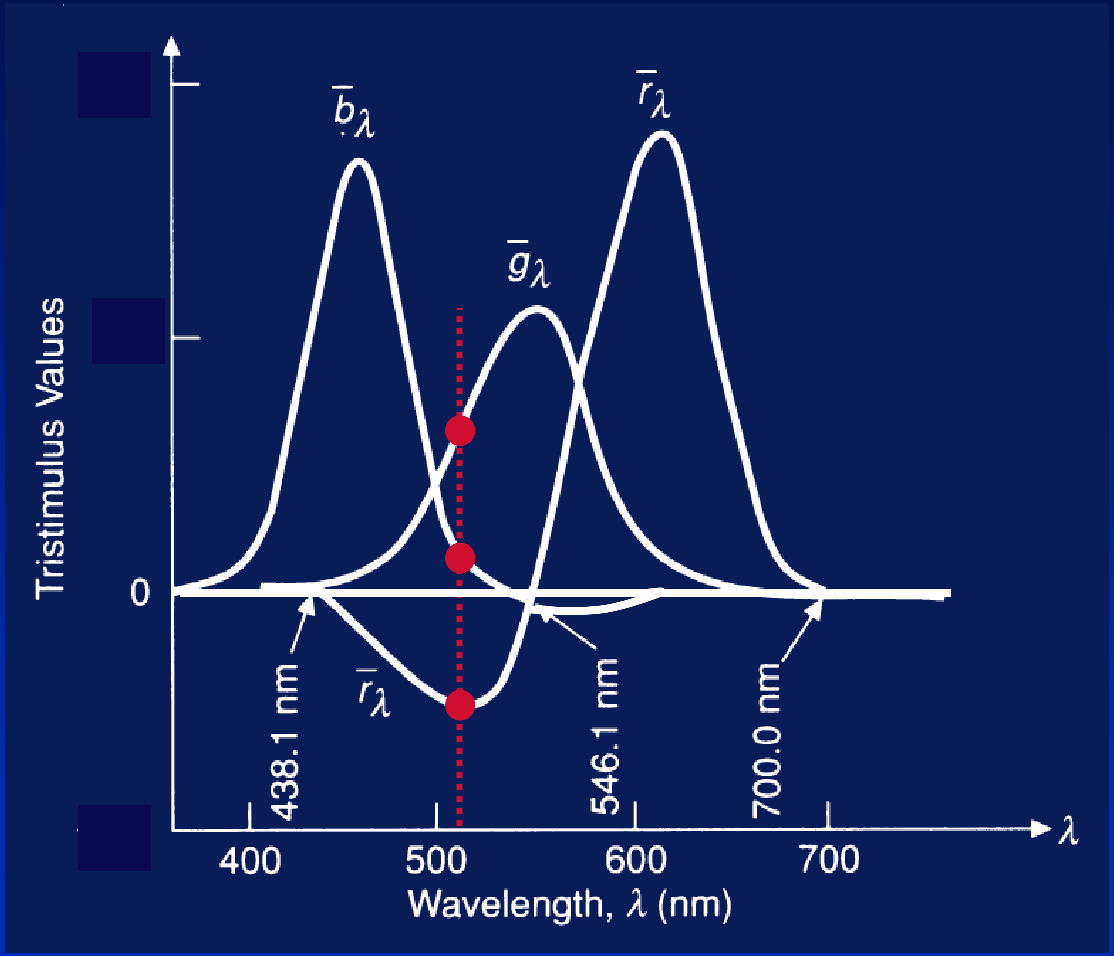


Experiment to Determine the Response Matching Functions of the Average Human Observer



Individually match the RGB primary lights to the unit values of each of the spectral lamps.

Response Matching Functions of the Average Human Observer



These are the response matching functions of the average human observer for these three primary lights.

Trichromatic Generalization

- Many colors can be matched by additive mixtures of suitable amounts of three fixed primary colors.
- Others have to be mixed with a suitable amount of one before it can be matched by the other two.
- All the colors can be matched in one of these two ways:
  - The restriction is that none of the primary colors can be matched by an additive mixture of the other two.

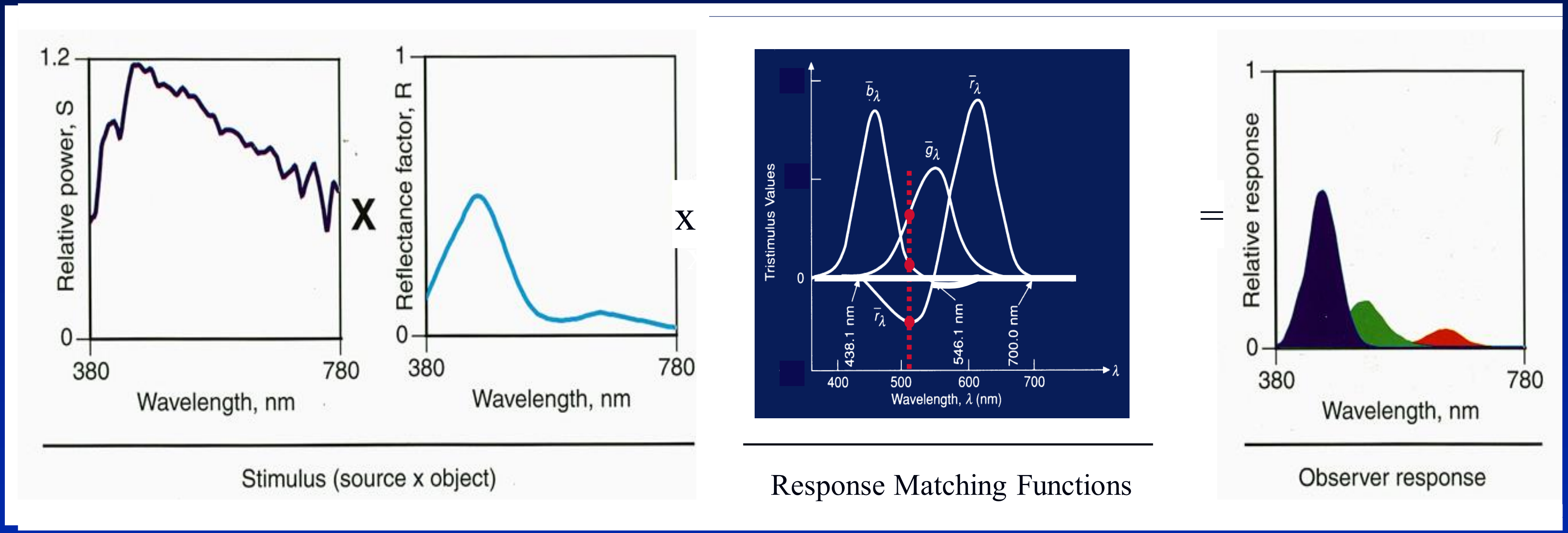
Trichromatic Generalization

– Proportionality and additivity are valid over a large range of observing conditions.

Proportionality - If A=B, then kA=kB

Additivity- If A=B, and C=D, the A+C=B+D

Observer Response



Roy S. Berns. "Billmeyer and Saltzman's Principles of Color Technology, 3rd Ed. 2000, John Wiley & Sons, Inc. p. 46.

Computing Tristimulus Values with the Response Matching Functions

- For each test lamp we can compute the equivalent RGB tristimulus values using the color matching functions

$$R = \int P(\lambda)\bar{r}(\lambda)d\lambda$$

$$G = \int P(\lambda)\bar{g}(\lambda)d\lambda$$

$$B = \int P(\lambda)\bar{b}(\lambda)d\lambda$$

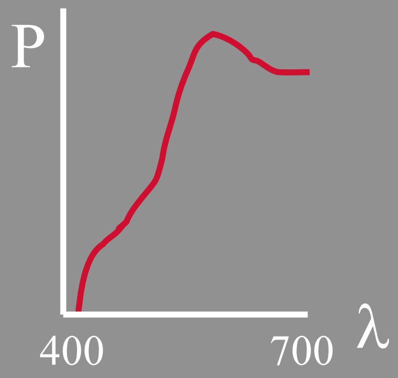
Comments on Response Matching Functions

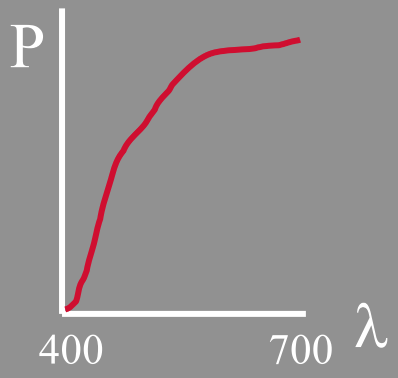
- Note that knowing the spectral distributions of the primary light sources and the response matching functions of the average human observer, we can then represent a large range of colors with any three primary light sources. (Monitors, cell phones, and printers).
- Perhaps most important, is the fact that these are perceptual spaces because a human observer is within the experimental testing loop.
- All of these tests have been conducted in a dark room and thus do not consider the effect of the illumination from the external environment.

Metamer

A metamer is a phenomenon
in which two spectrally different stimuli
match to a given observer.

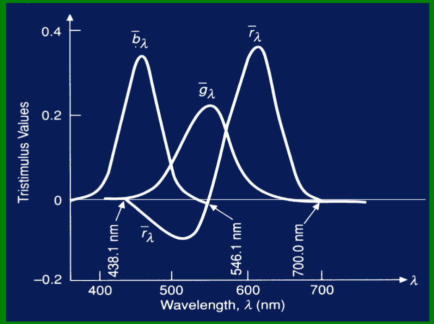
Metamers

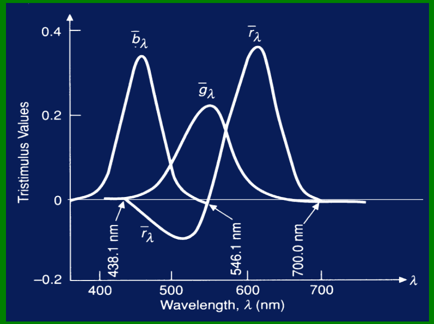




Physically different

×   ×





Perceptual filter

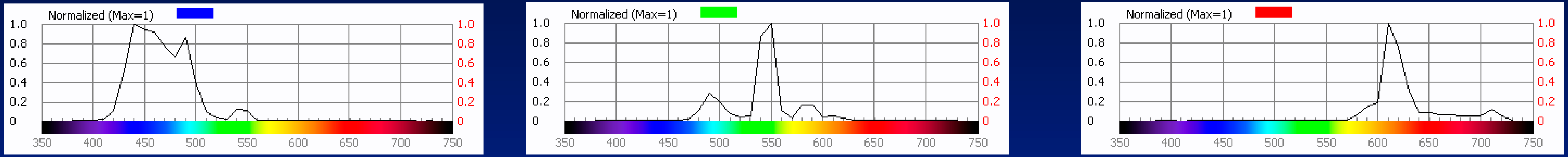
R,G,B = *R,G,B*

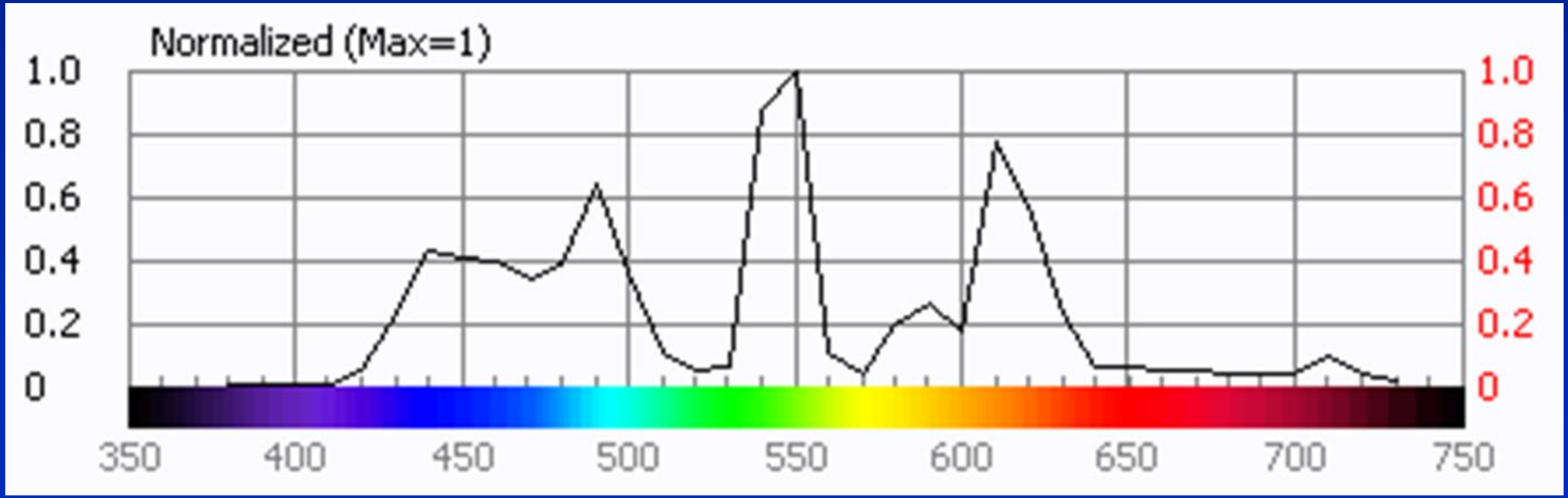
Perceptually equal

Metamers

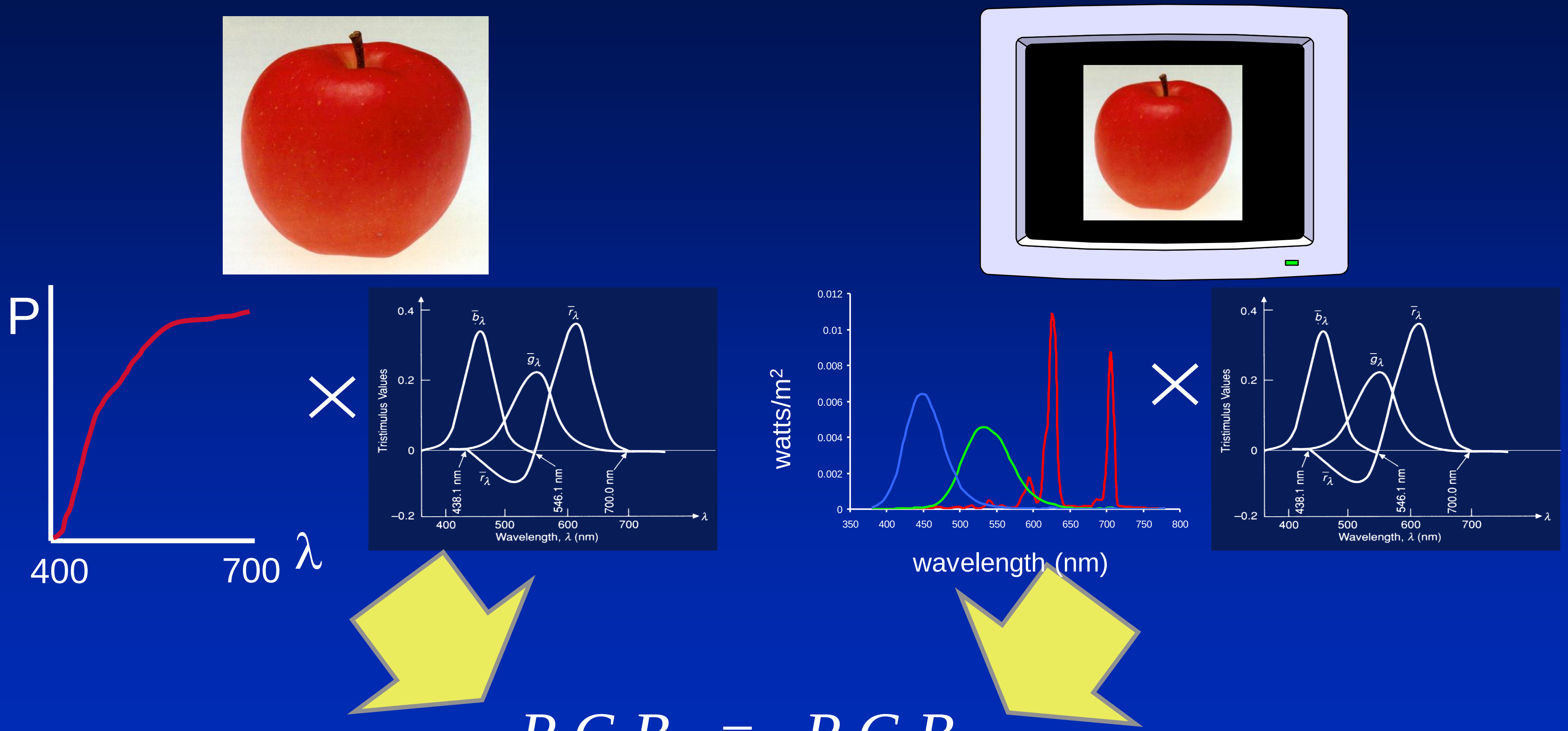
- Trichromacy, perhaps the most important property of the visual system, leads to metamerism. (Grassman 1853)
- Metamerism is a phenomenon in which spectrally different stimuli match to a given observer.
- Because of metamerism, color reproduction is possible.
- Stimuli do not have to have identical spectral properties in order to have a perceptual color match.

Typical LCD spectral radiance distribution





Taking Advantage of Metamers

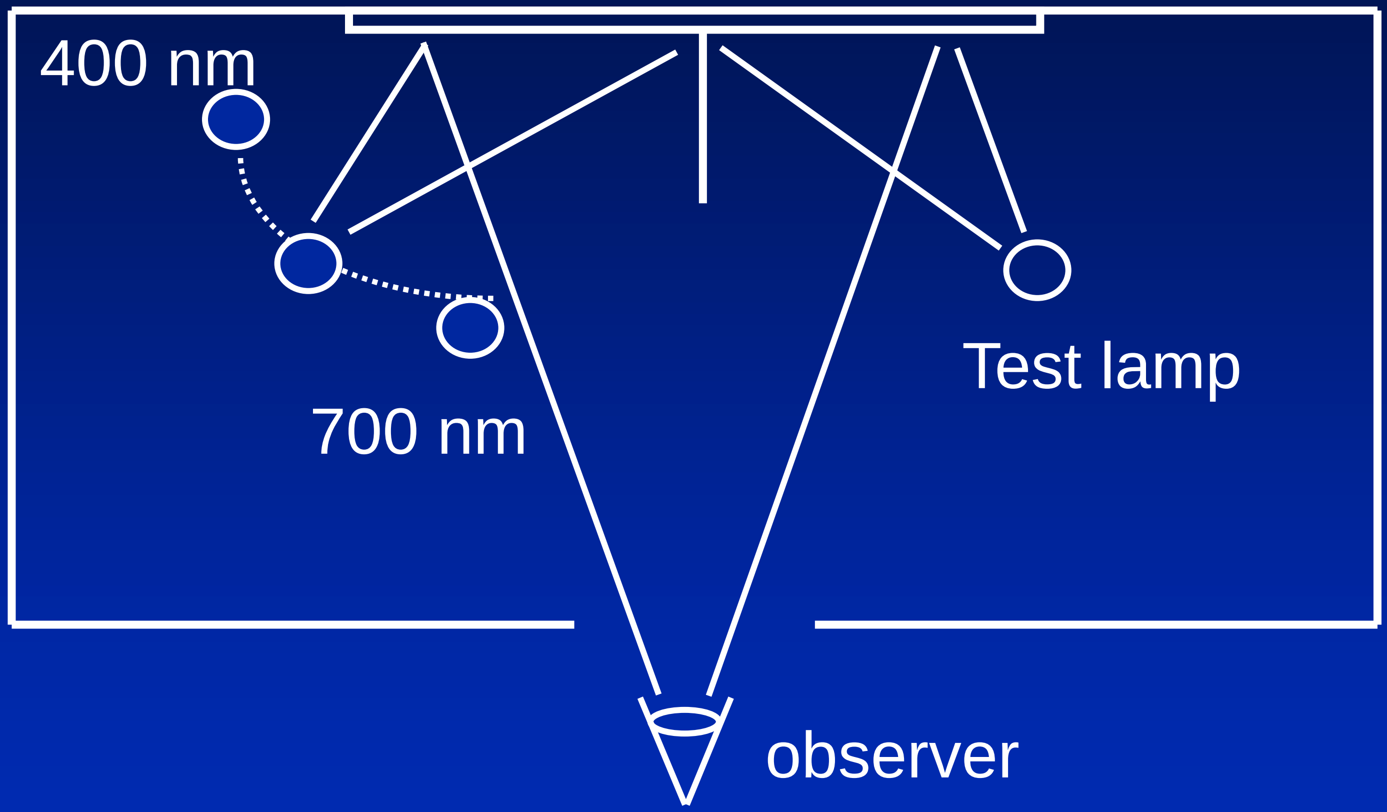


$$R,G,B \quad = \quad R,G,B$$

Visual match

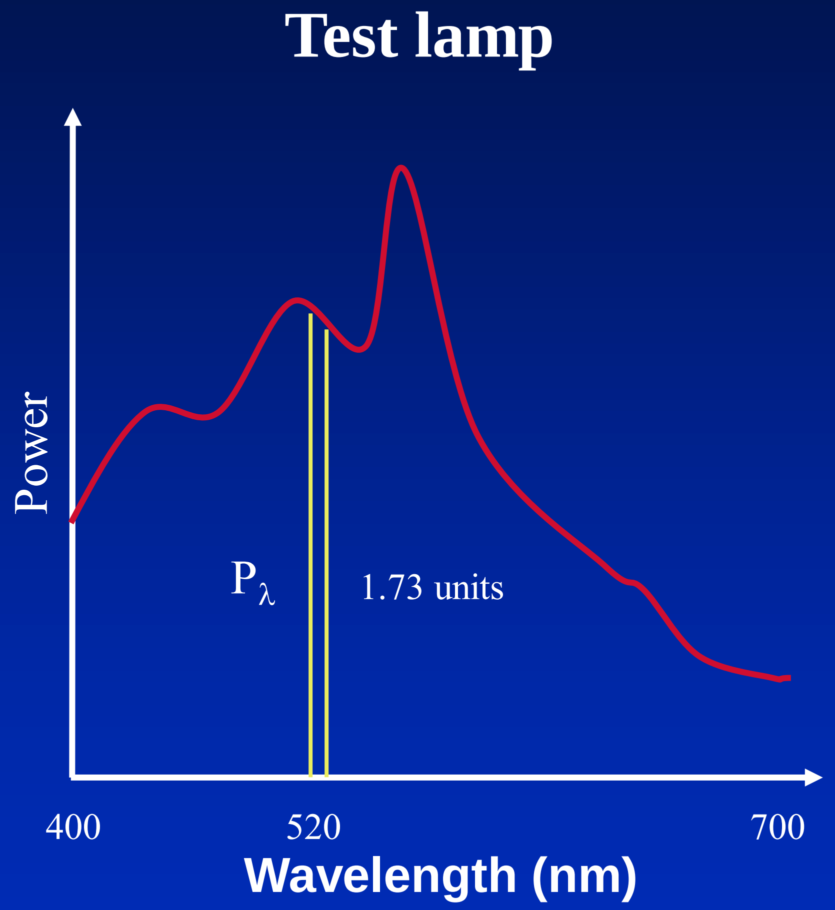
A New Experiment to Match the Power Spectrum of a Test Lamp

Matching the Power Spectrum of the Test Lamp with a Full Set of Spectral Lights



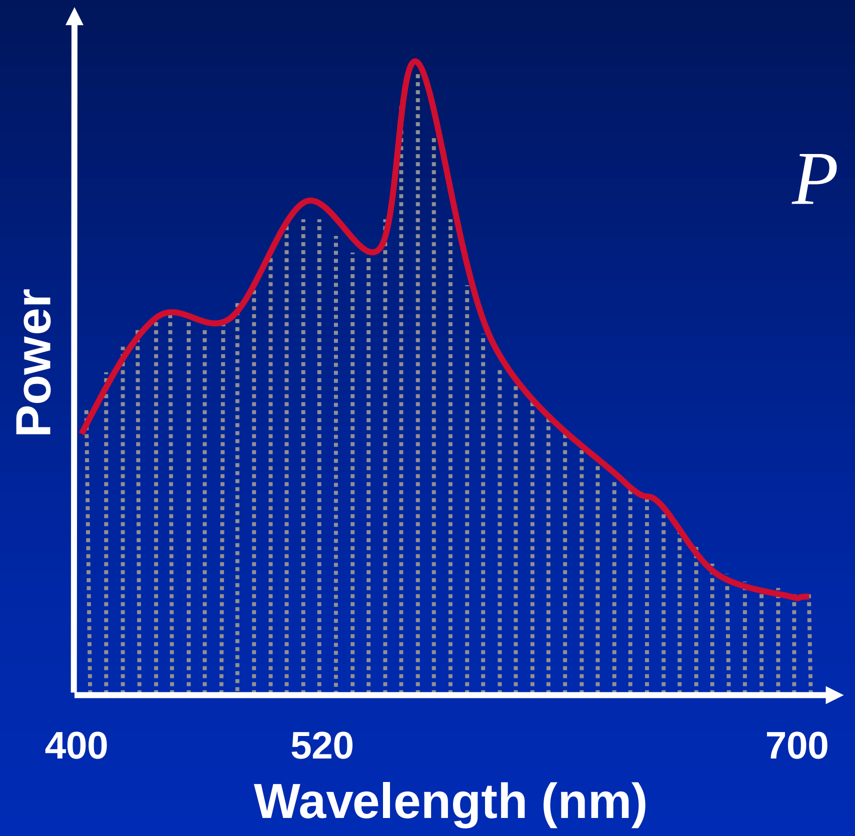
Energy of each of the spectral lights is adjusted to match a test lamp with its power spectrum

Spectral Distribution of Test Lamp



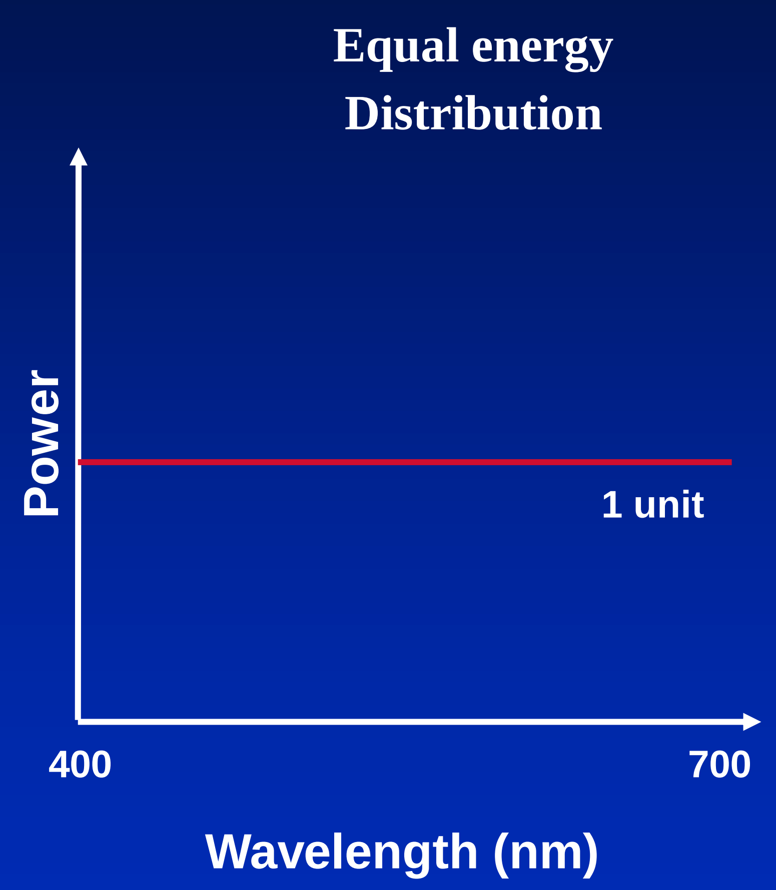
Energy of each of the spectral lights is adjusted to match a test lamp with its power spectrum

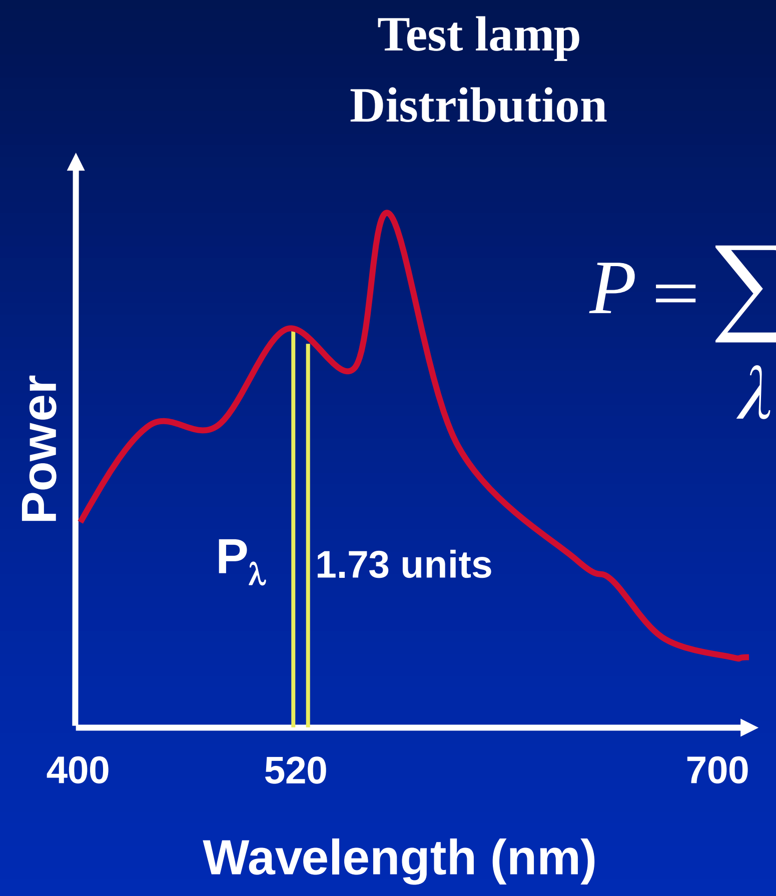
Matched Spectral Distribution of Test Lamp



$$P = \sum_{\lambda} P_{\lambda}$$

Matched Spectral Distribution of Test Lamp

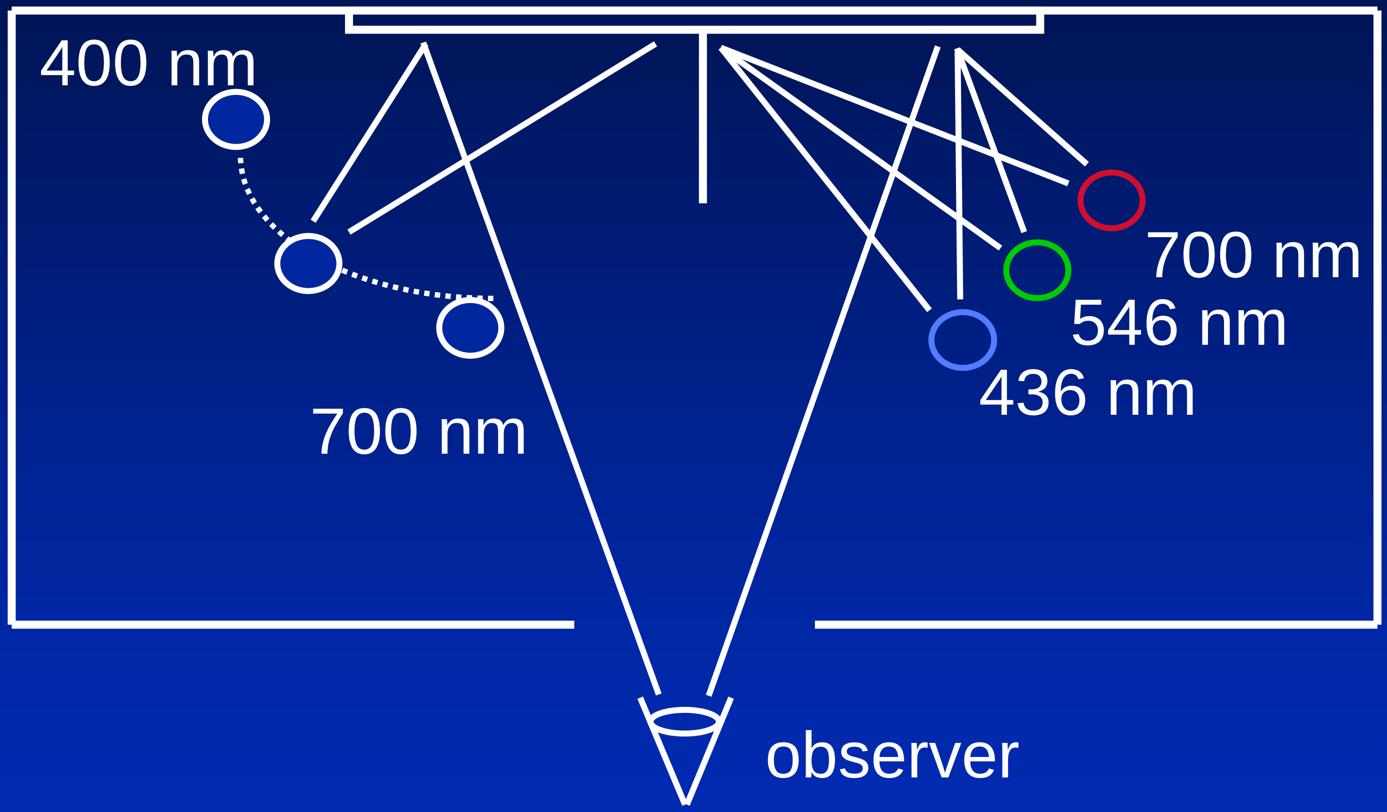




$$P = \sum_{\lambda} P_\lambda$$

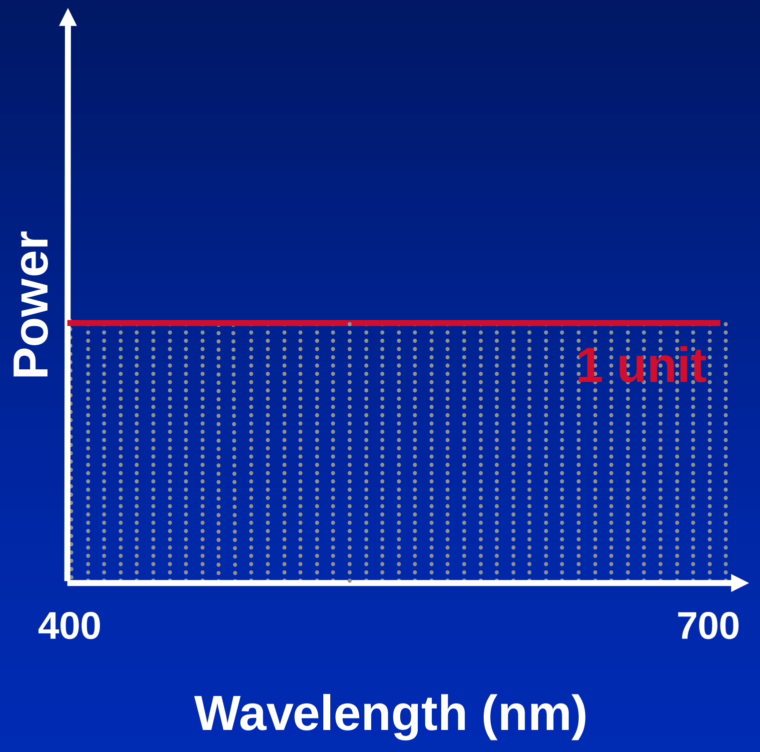
To get spectrum light energy of test lamp, need to multiply each unit energy spectral light by P_λ .

Matching the Power Spectrum of the Test Lamp with 3 Primary Lights



Matching the equivalent of the test lamp's power spectrum with three primary lights.

Equal Energy Distribution

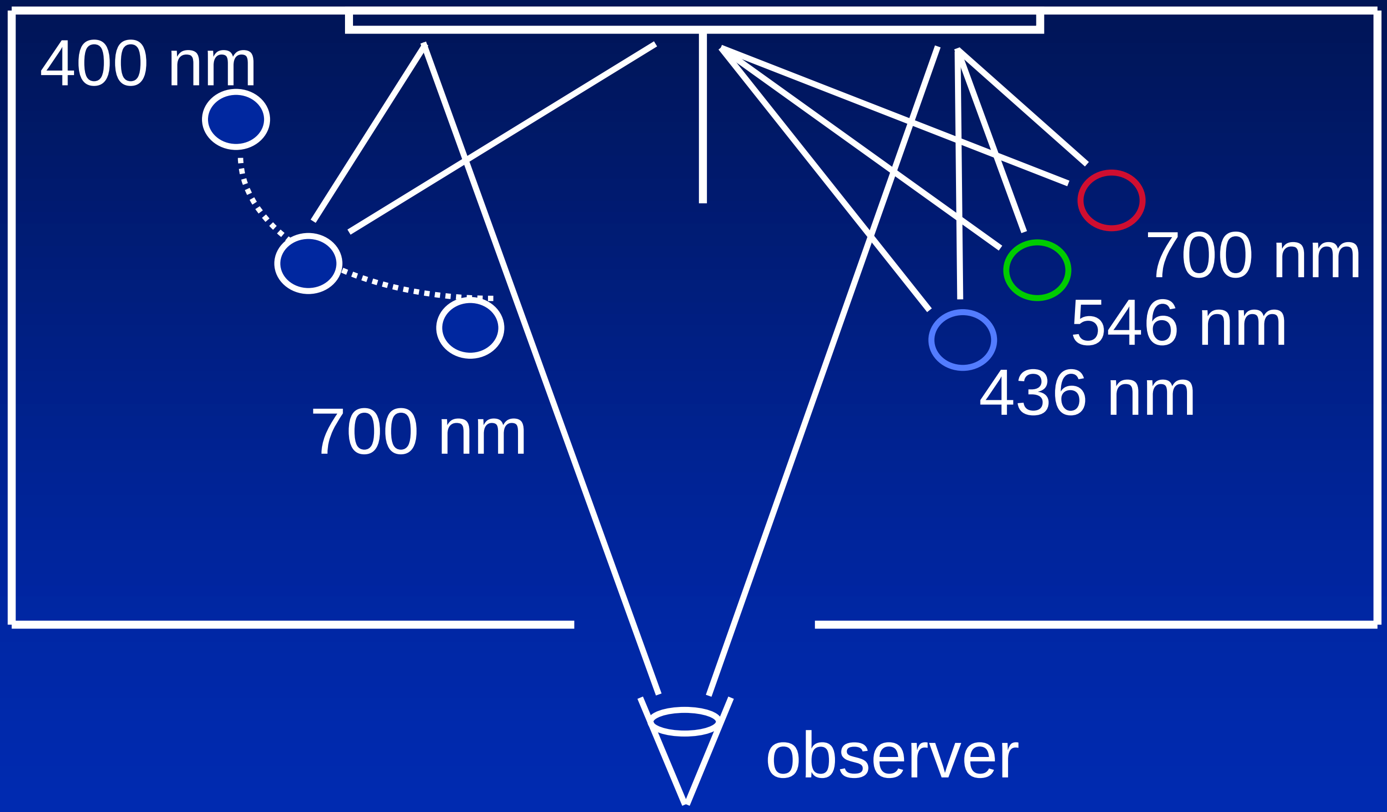


From the response matching functions,

$$P_\lambda = 1 = r_\lambda + g_\lambda + b_\lambda$$

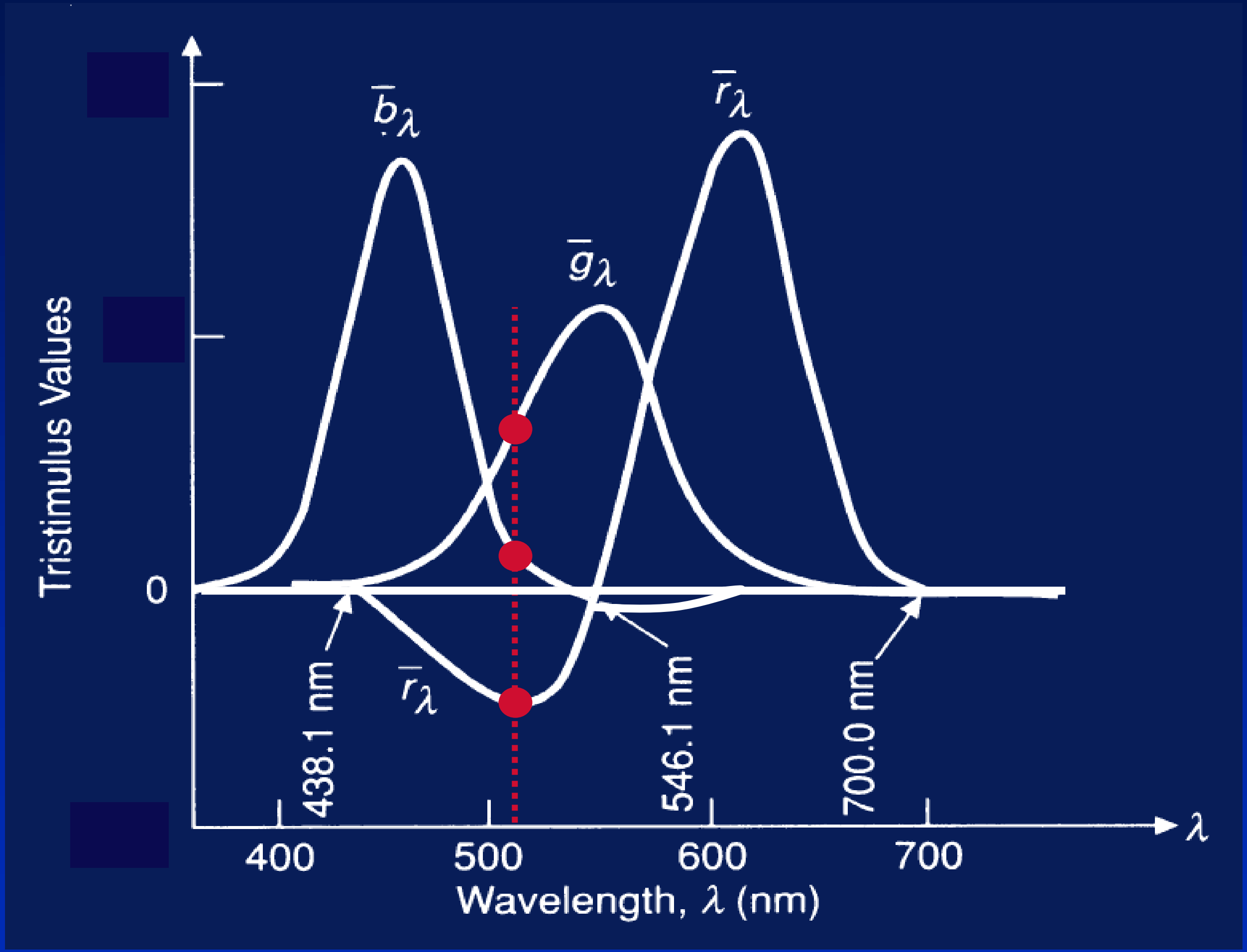
for each wavelength.

Response Matching Functions



Individually match the RGB primary lights to the unit values of each of the spectral lamps.

Response Matching Functions

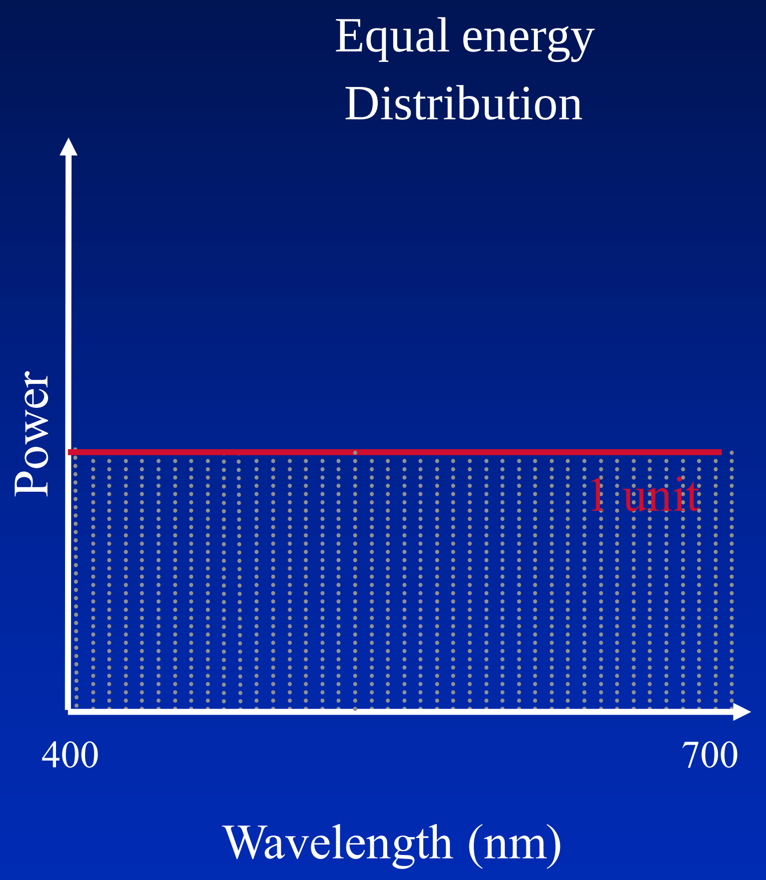


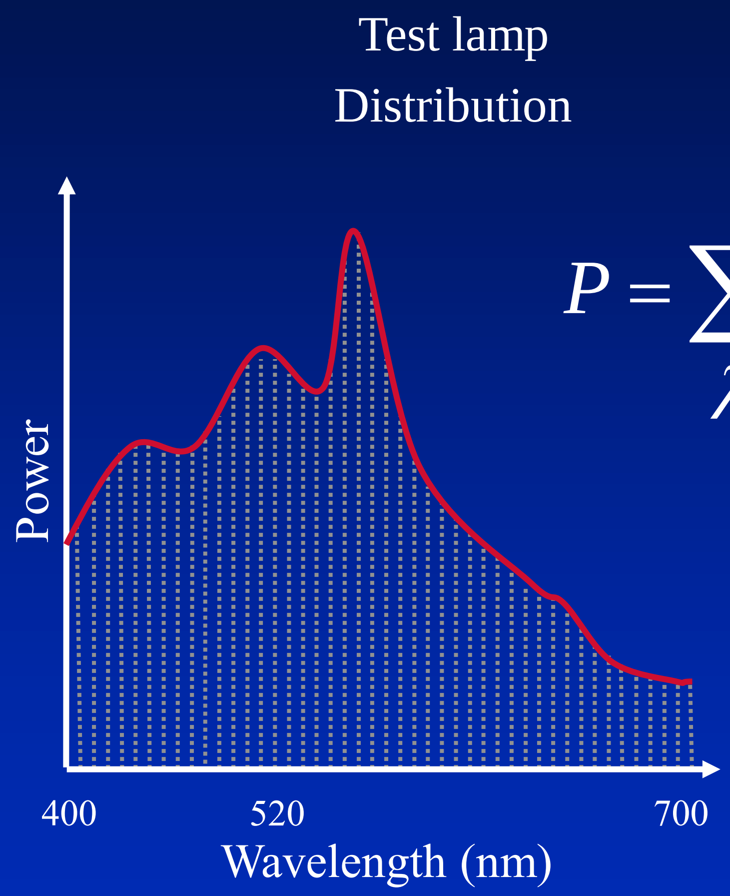
These are the response matching functions of the average human observer for these three primary lights.

If we cannot conclusively establish physiological properties of the human eye, how can we quantitatively work with color?

Answer: We treat the human visual system as a black box, and characterize it by its response to color stimuli.

Spectral Distribution of Test Lamp





$$P = \sum_{\lambda} P_{\lambda}$$

To get spectrum light energy of test lamp, need to multiply each unit energy spectral light by P_{λ} .

Computing Tristimulus Values

$$P = \sum_{\lambda} P_{\lambda} = \sum_{\lambda} P_{\lambda}\overline{r_{\lambda}} + P_{\lambda}\overline{g_{\lambda}} + P_{\lambda}\overline{b_{\lambda}}$$

$$\boldsymbol{R = \sum_{\lambda} P_{\lambda}\overline{r_{\lambda}} \quad , \quad G = \sum_{\lambda} P_{\lambda}\overline{g_{\lambda}} \quad , \quad B = \sum_{\lambda} P_{\lambda}\overline{b_{\lambda}}}$$

The energy of the entire test lamp, P, is equal to the sum of the individual wavelength energies.

Computing Tristimulus Values with the Response Matching Functions

- For each test lamp we can compute the equivalent RGB tristimulus values using the color matching functions

$$R = \int P(\lambda)\bar{r}(\lambda)d\lambda$$

$$G = \int P(\lambda)\bar{g}(\lambda)d\lambda$$

$$B = \int P(\lambda)\bar{b}(\lambda)d\lambda$$

So what?

What have we done?

What is its value?

- Uncle Don

So what?

- In the first test, for a given test lamp we had to experimentally find the RGB values of the primary lights.
- In the second test, we experimentally found the response matching functions of the average human observer.

So what?

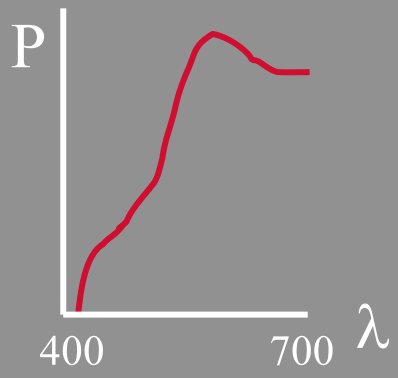
- In the third test, if we know the spectral distribution of the test lamp, and we know the response matching functions of the average human observer, we can **PREDICT** the RGB values of the primary lights.

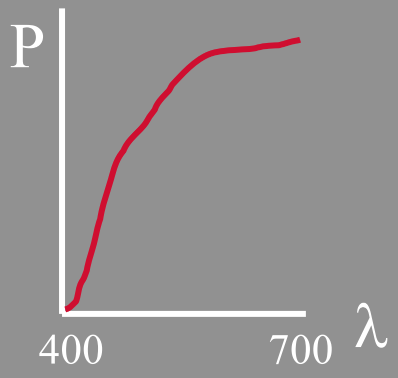
We do not have to perform the experiments.

Metamer

A metamer is a phenomenon
in which two spectrally different stimuli
match to a given observer.

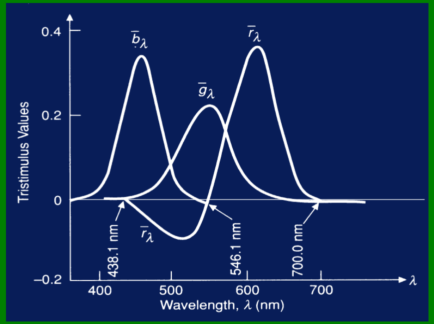
Metamers

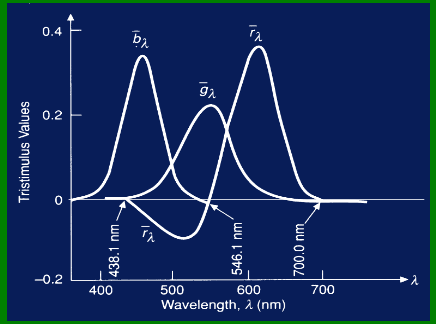




Physically different

×   ×





Perceptual filter

$R,G,B \quad = \quad R,G,B$

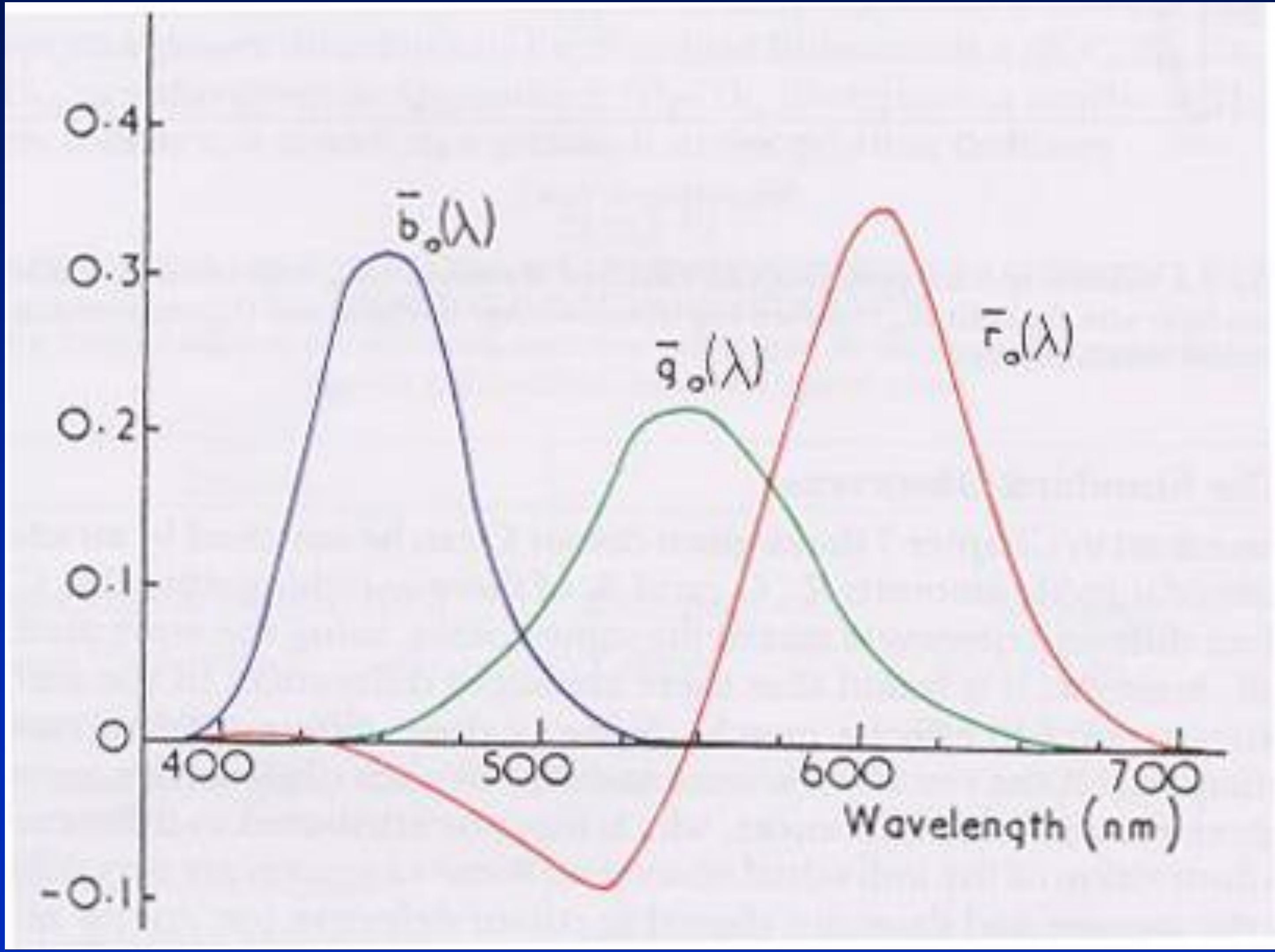
Perceptually equal

What happens if we have a different set of three primary lights?

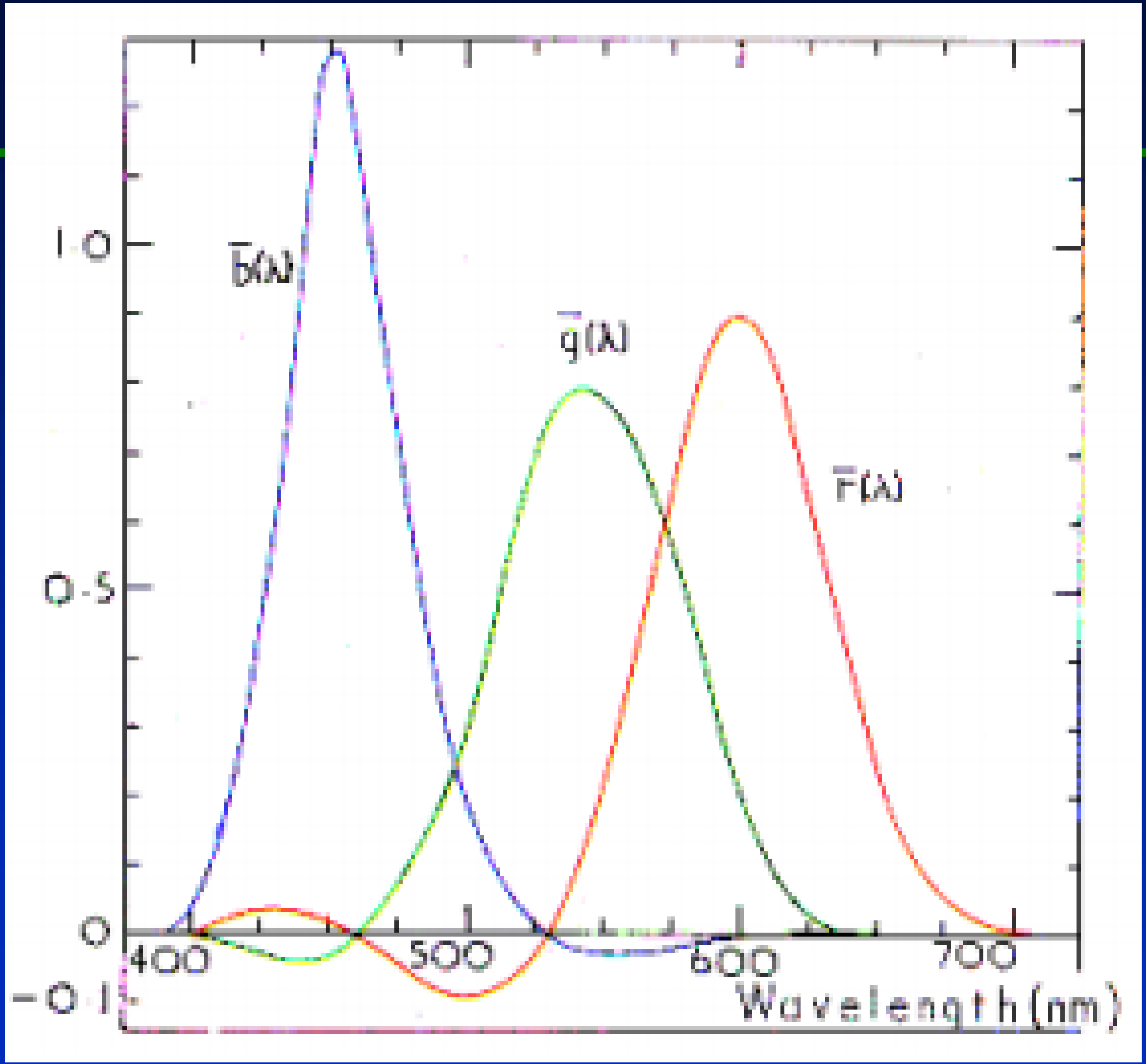
We get a different set of response matching functions

Response Matching Functions

R = 700 nm
G = 546.1 nm
B = 435.8 nm



R = 650 nm
G = 530 nm
B = 460 nm



Transformations

- A new set of response matching functions is generated for each new set of primary colors.

- It is possible to mathematically transform the response matching functions for each new set of primary colors.

- It is also possible to mathematically transform the tristimulus values from one set of primaries to another.

Transformation of Color Primaries

$$R' = a_{11}R + a_{12}G + a_{13}B$$
$$G' = a_{21}R + a_{22}G + a_{23}B$$
$$B' = a_{31}R + a_{32}G + a_{33}B$$

$$\begin{Bmatrix} R' \\ G' \\ B' \end{Bmatrix} = [\,\mathrm{A}\,] \begin{Bmatrix} R \\ G \\ B \end{Bmatrix}$$

New color primary lights can be matched by the original color primary lights. These are the new tristimulus values.

Comments on Response Matching Functions

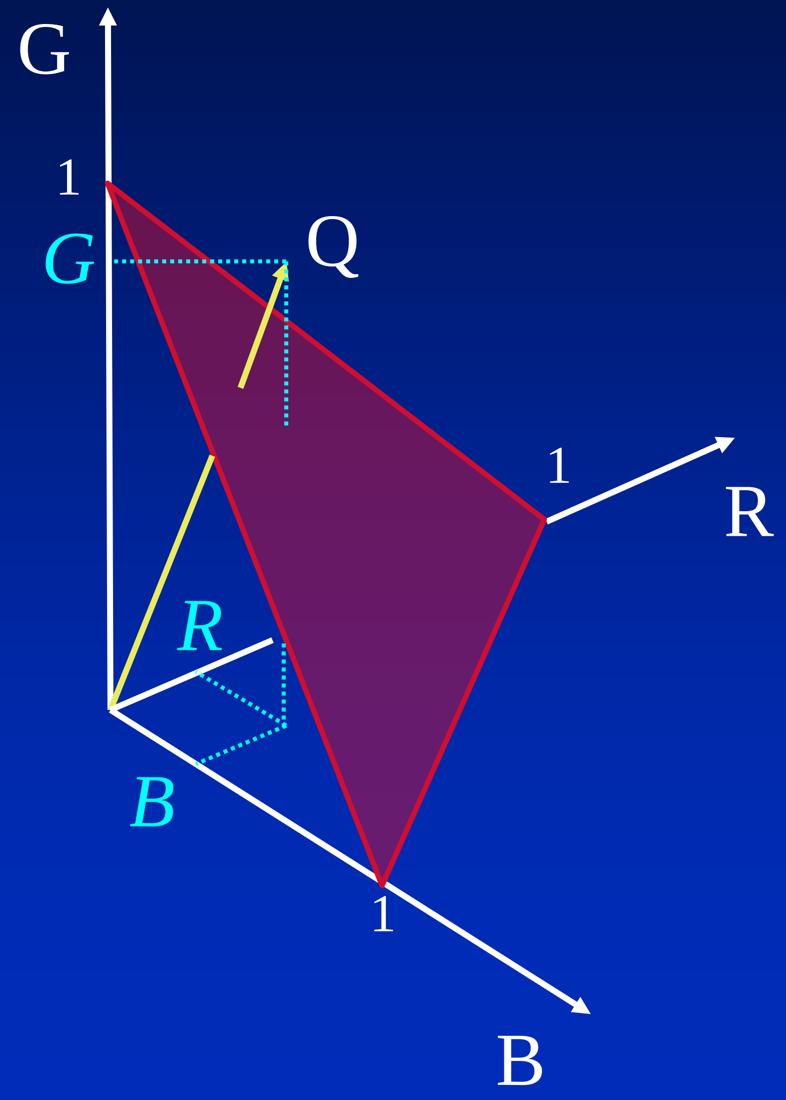
- Note that knowing the spectral distributions of the primary light sources and the response matching functions of the average human observer, we can then represent a large range of colors with any three primary light sources. (Monitors, cell phones, and printers).
- Perhaps most important, is the fact that these are perceptual spaces because a human observer is within the experimental testing loop.
- All of these tests have been conducted in a dark room and thus do not consider the effect of the illumination within the external environment.

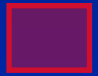
Chromaticity Diagrams

How can we describe the perceptual color space in a way which is easy to use and consistent with its results no matter what input or output devices are used?

- Television (video)
- Digital cameras
- Printers
- Projectors

RGB Space

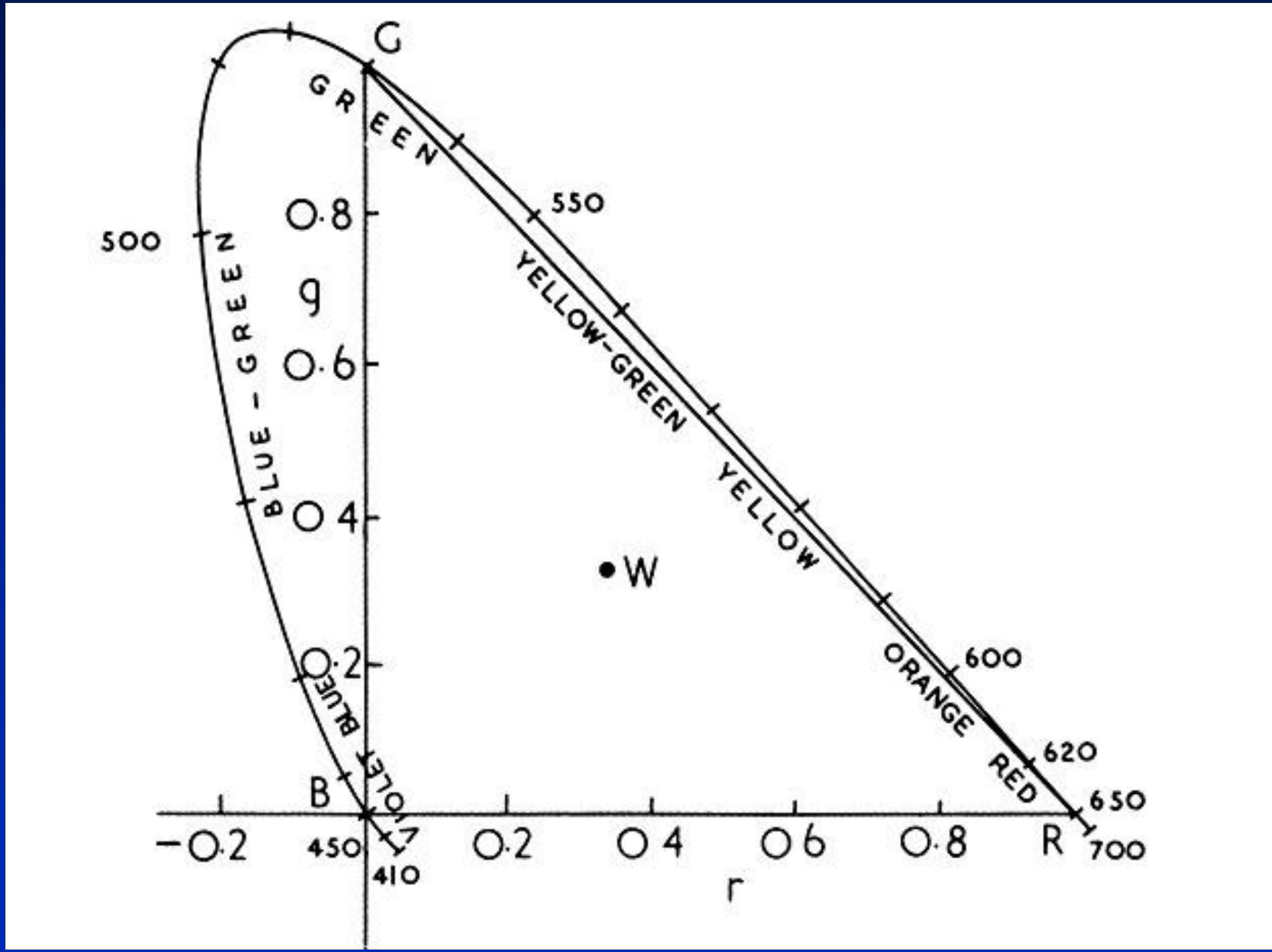


- Q: Color vector
- *R*, *G*, *B*: tristimulus values
  i.e. RGB coordinates
-  :Unit plane ($r+g+b = 1$)

Color Triangle for RGB Primaries

R = 650 nm
G = 530 nm
B = 460 nm

r+g+b = 1



Color Triangle for RGB Primaries

R = 650 nm
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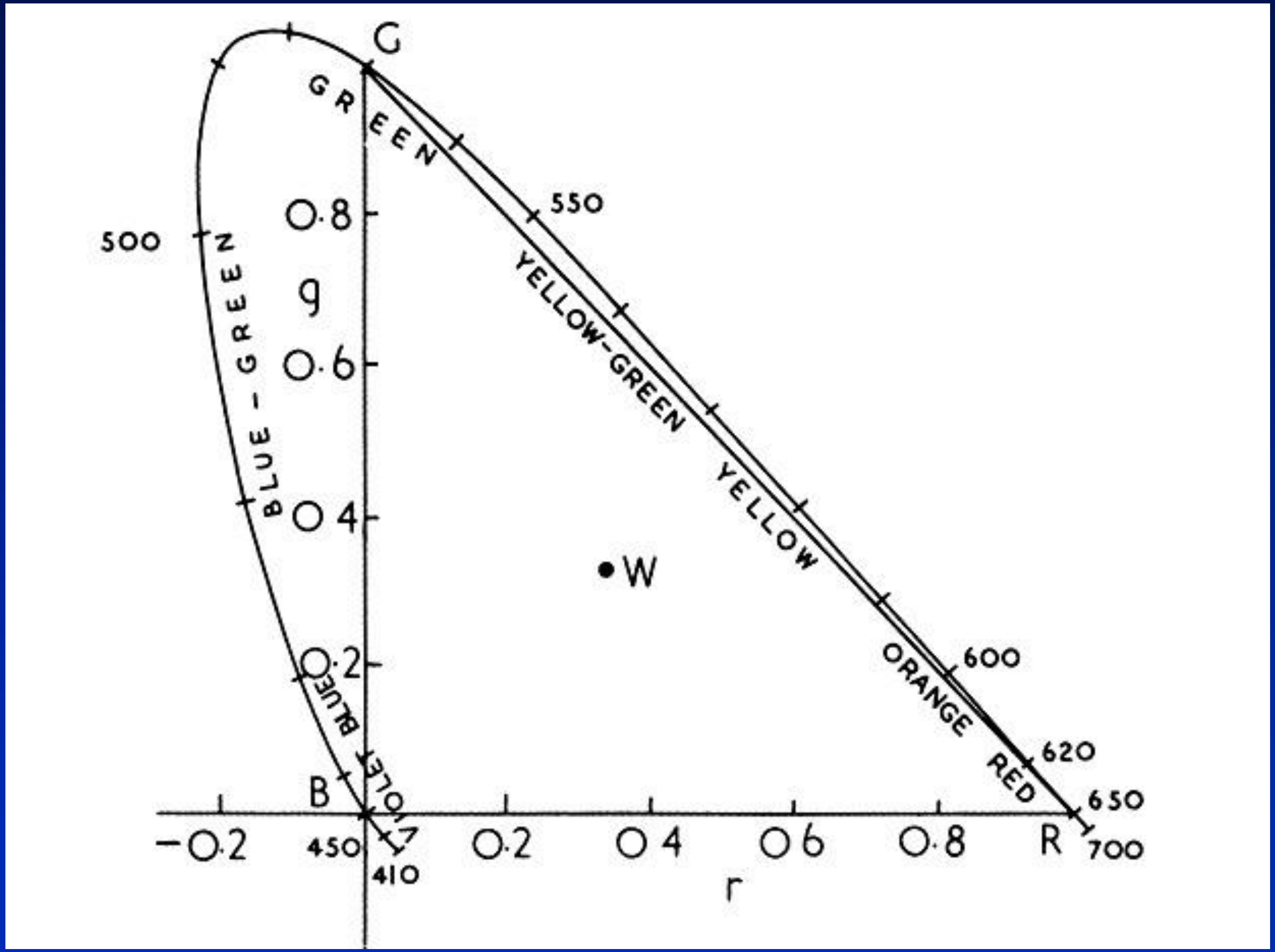

Note: **Only colors lying within the color triangle rgb can be exactly reproduced.**

Response Matching Functions

R = 700 nm
G = 546.1 nm
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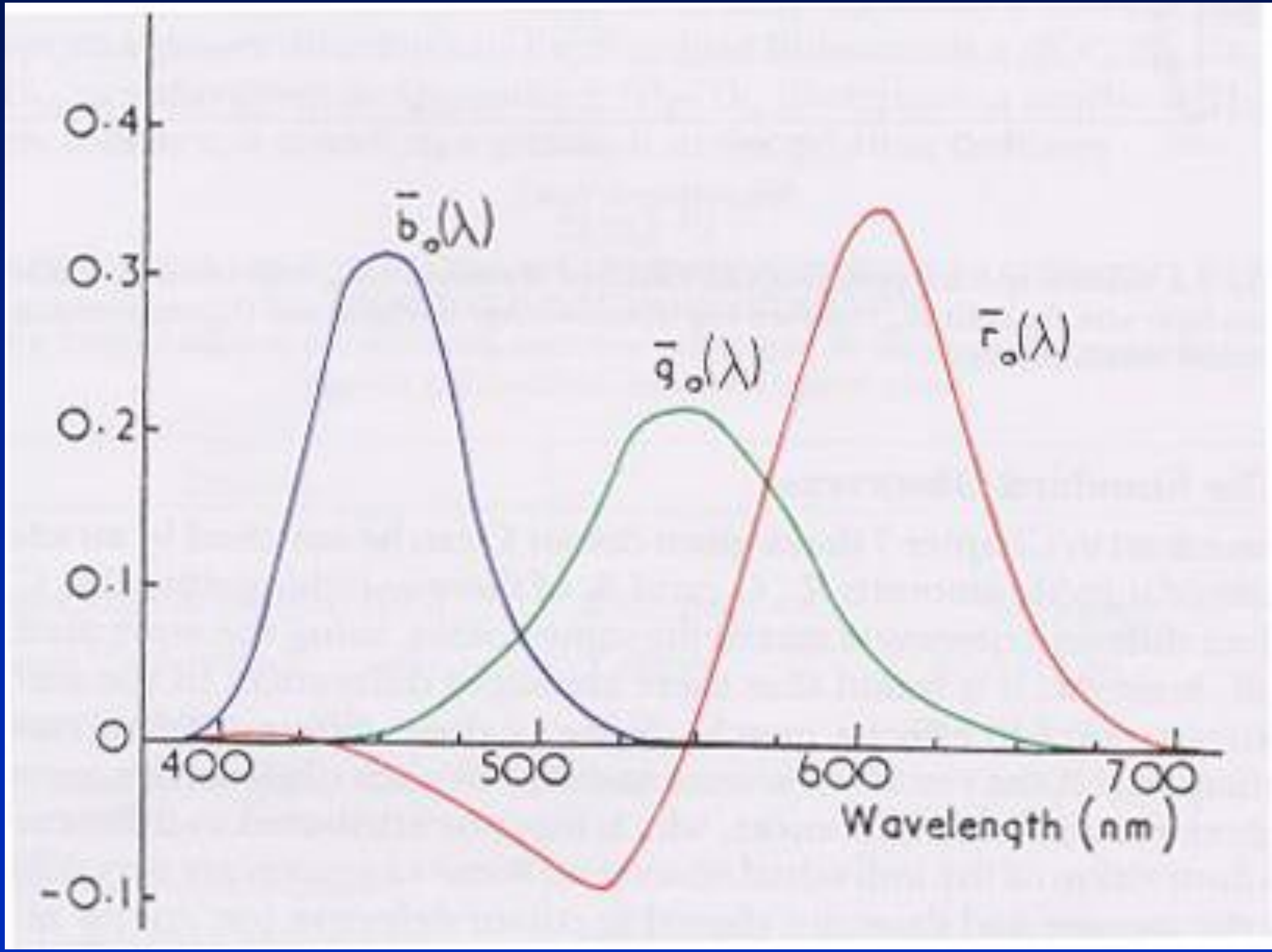


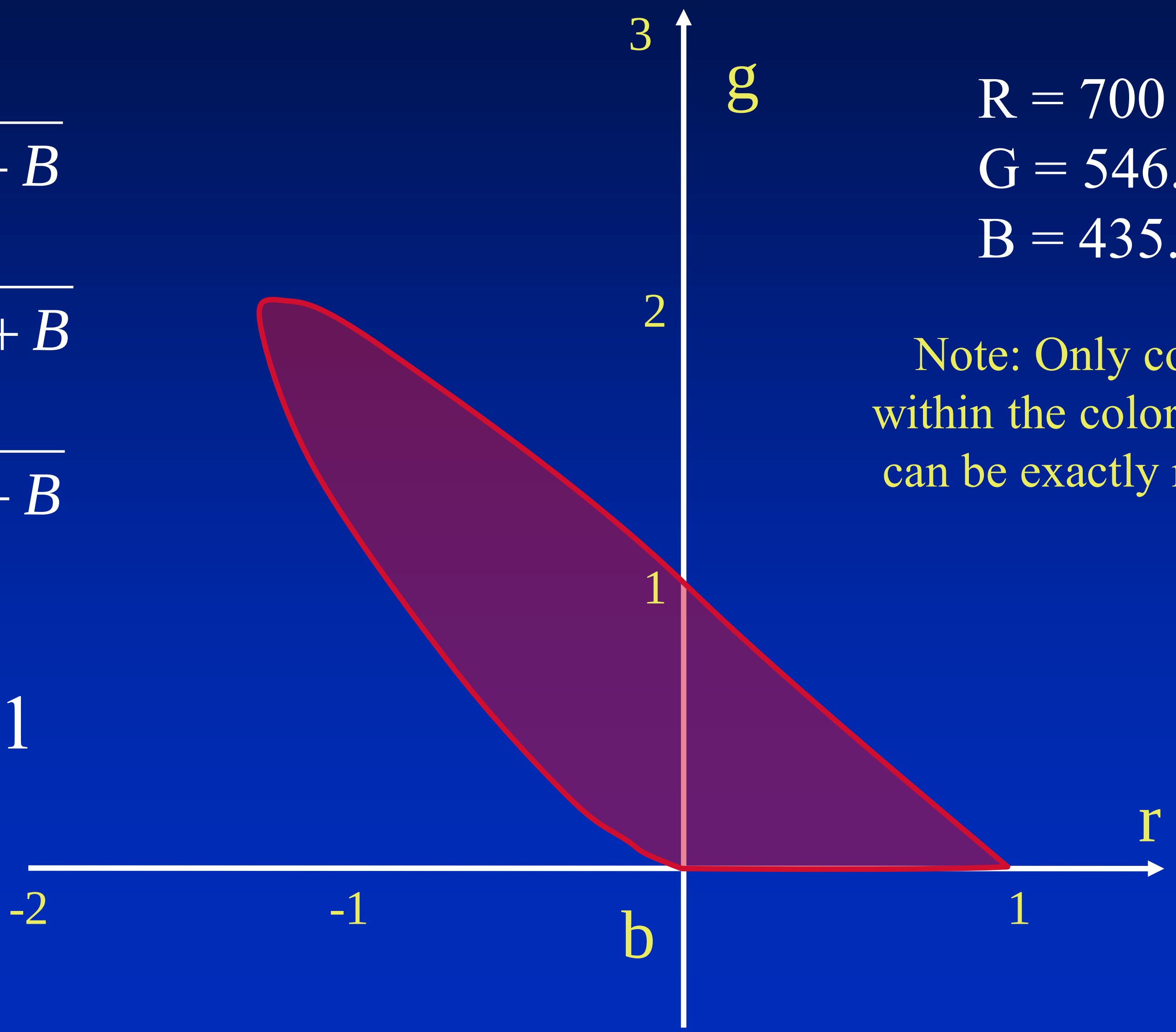
Color Triangle for RGB Primaries

$$r = \frac{R}{R+G+B}$$

$$g = \frac{G}{R+G+B}$$

$$b = \frac{B}{R+G+B}$$

$$r+g+b = 1$$



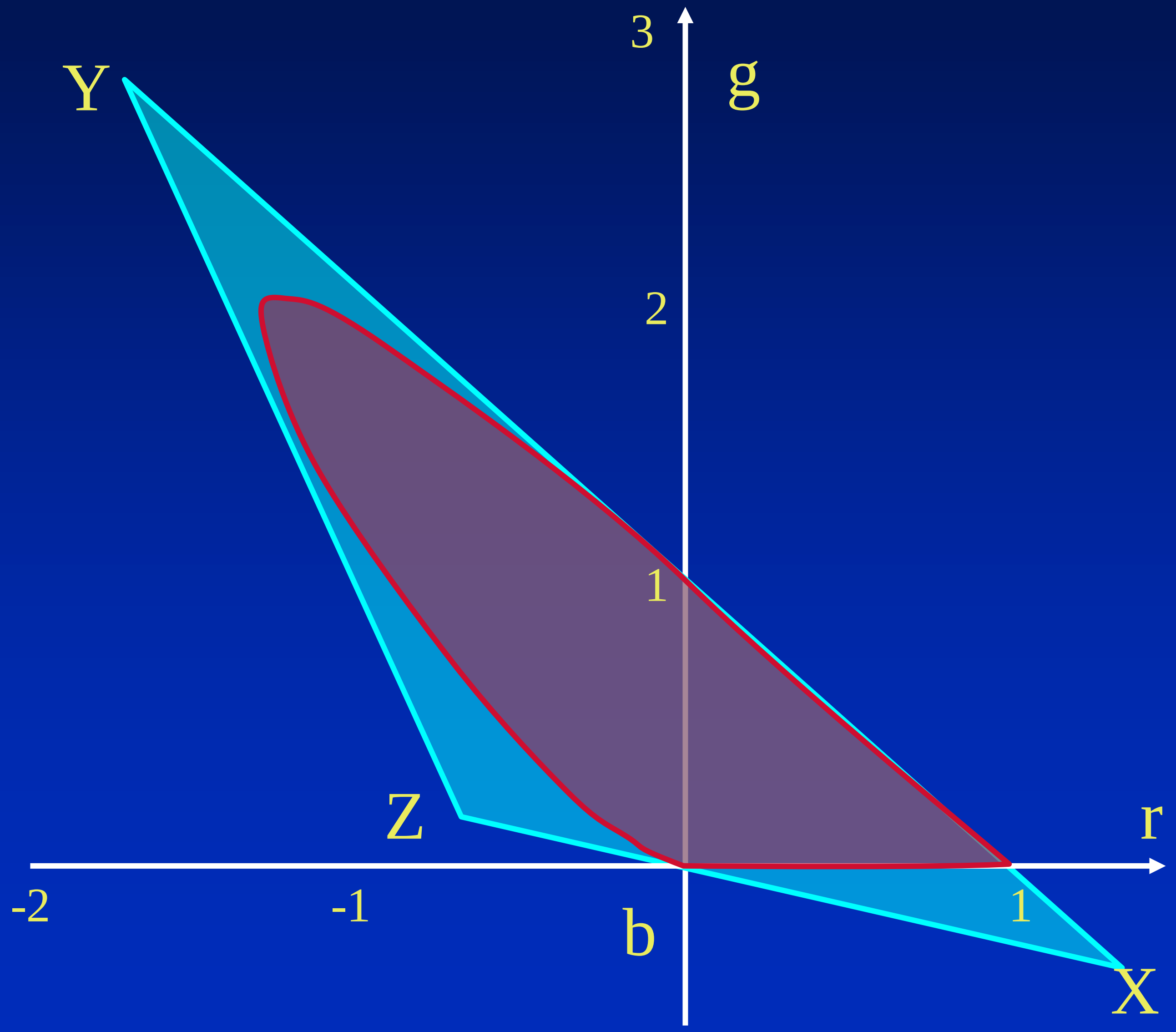
R = 700 nm
G = 546.1 nm
B = 435.8 nm

Note: Only colors lying within the color triangle rgb can be exactly reproduced.

Commentary

- Dealing with color spaces that have negative regions is mathematically difficult and not intuitive
- How can we rectify this?

Color Triangle for RGB and XYZ



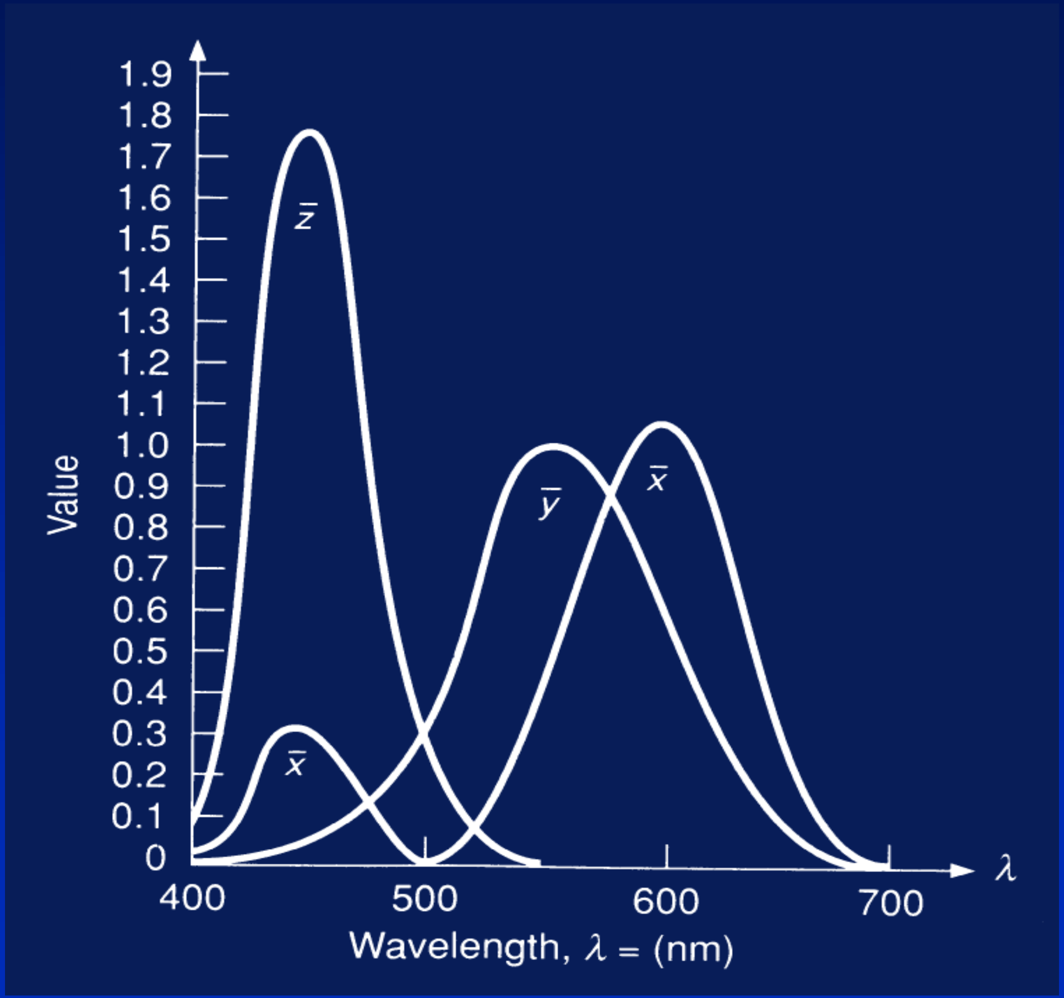
XYZ Assumptions

1. One coordinate, and one coordinate only, should represent the luminance.
2. The line between x and y should be nearly coincident with the spectral locus for colors in the 550nm to 700nm (green-red) range.
3. The spectral locus of all realizable colors should lie in the all-positive XYZ quadrant.

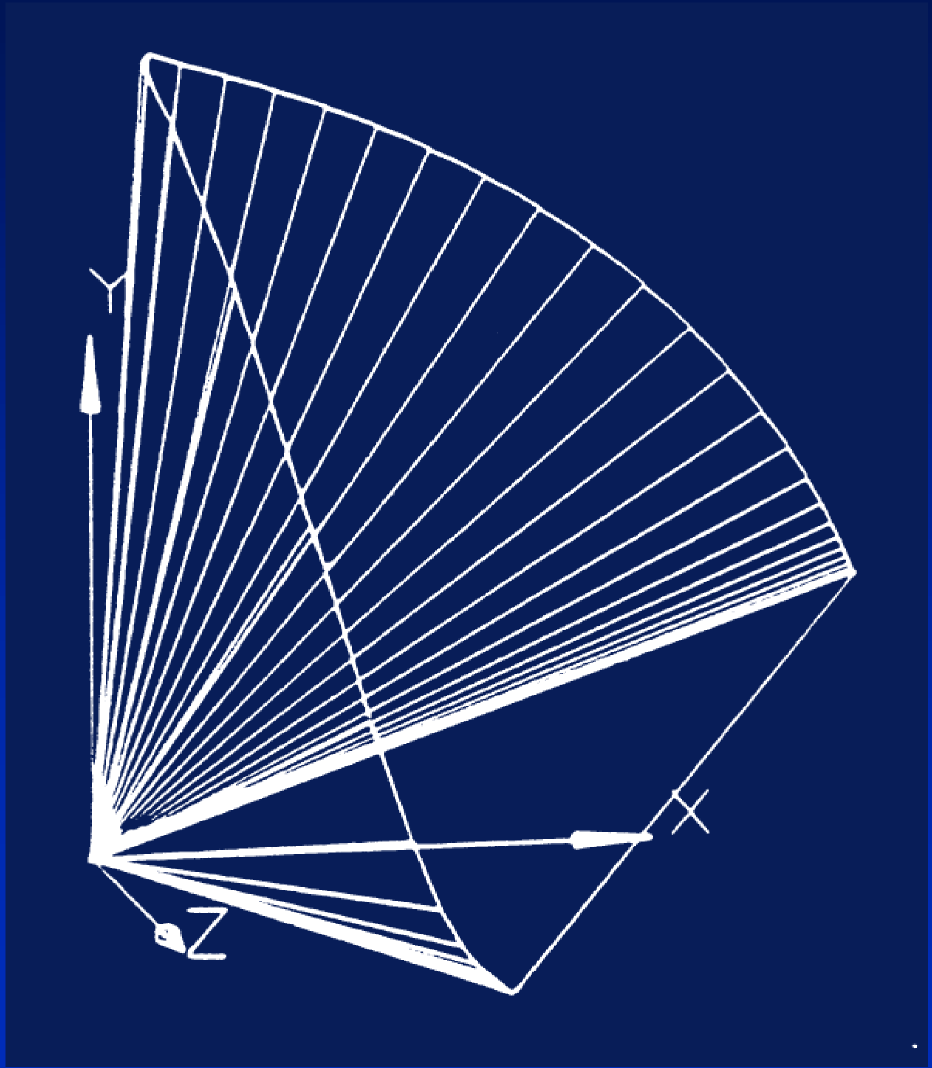
∴ For realizable colors, all XYZ values and all color matching functions are positive.

XYZ Color Matching Functions

- All positive values
- $\bar{y}$ is the luminous efficiency function
- Equal area



XYZ Color Space



Describing Color in XYZ

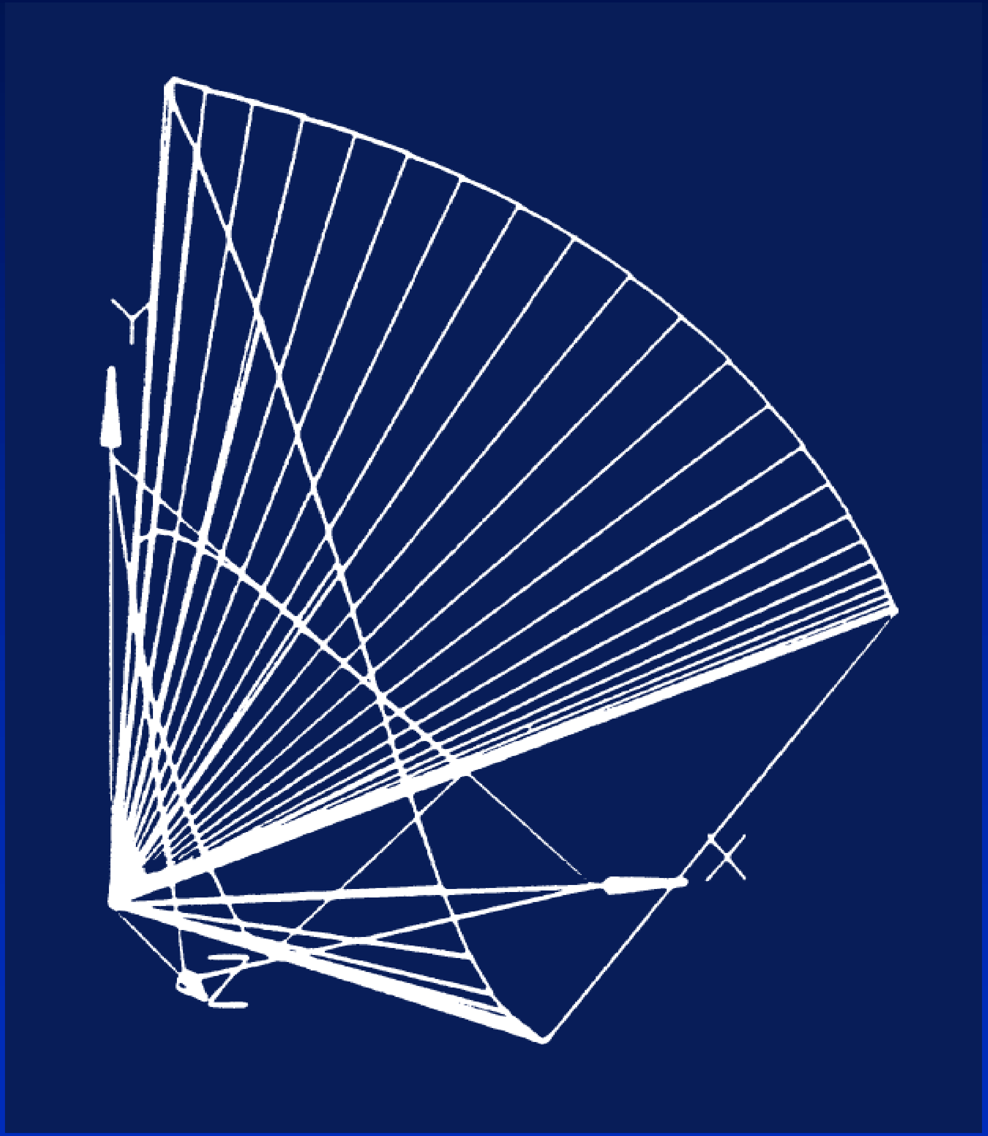
- Luminance Y
- Chromaticity

$$x = \frac{X}{X+Y+Z}$$

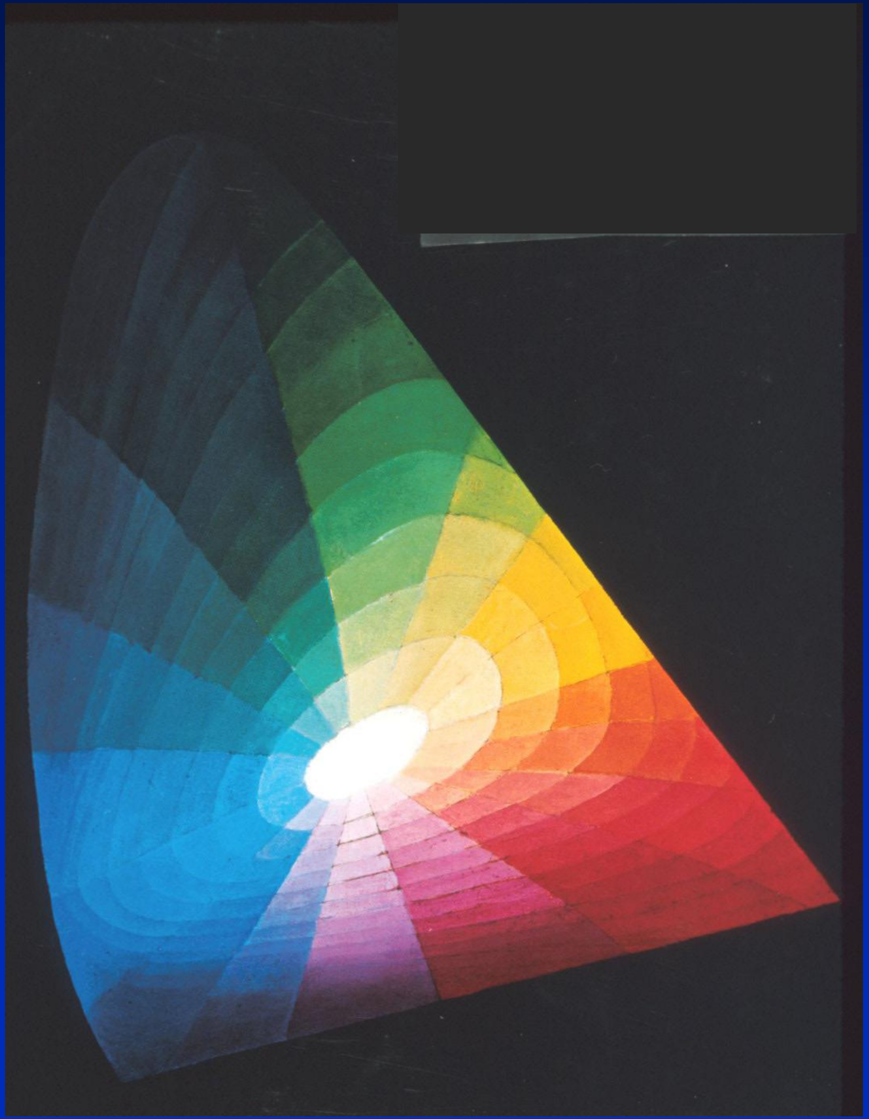
$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

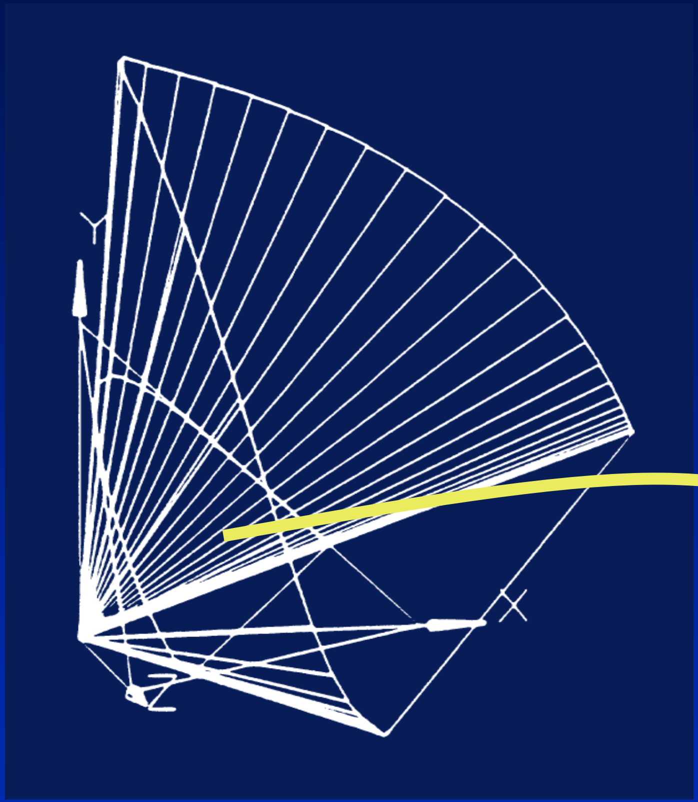
$$(x + y + z = 1)$$

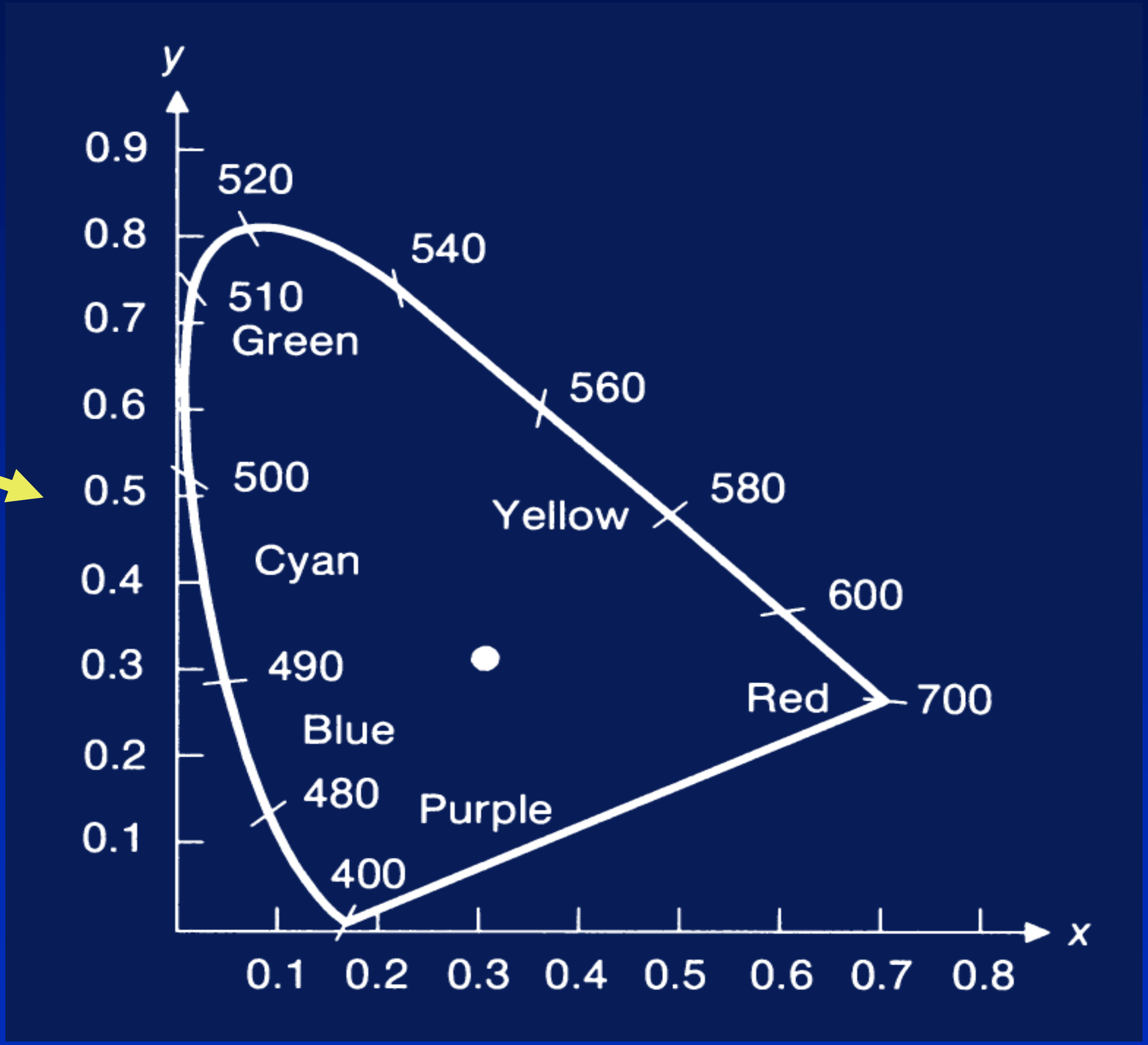


Chromaticity Diagram

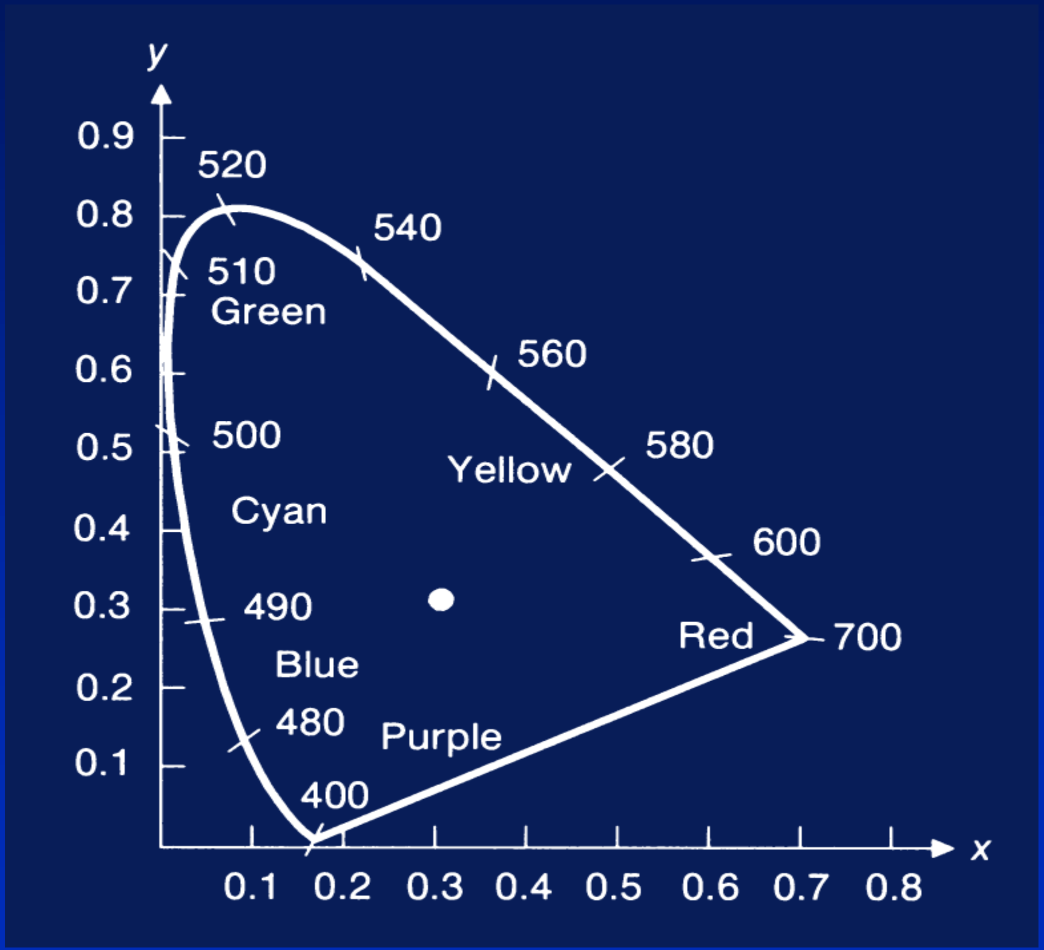


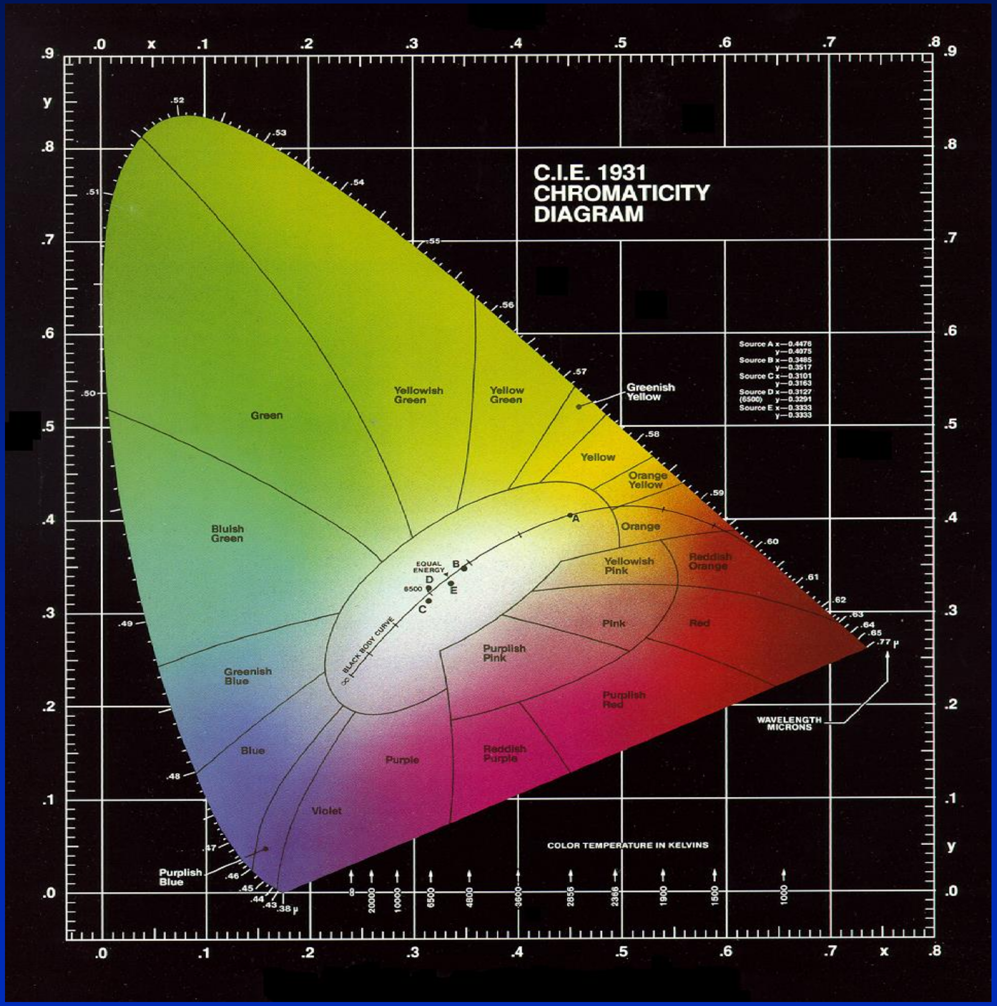
Chromaticity Diagram

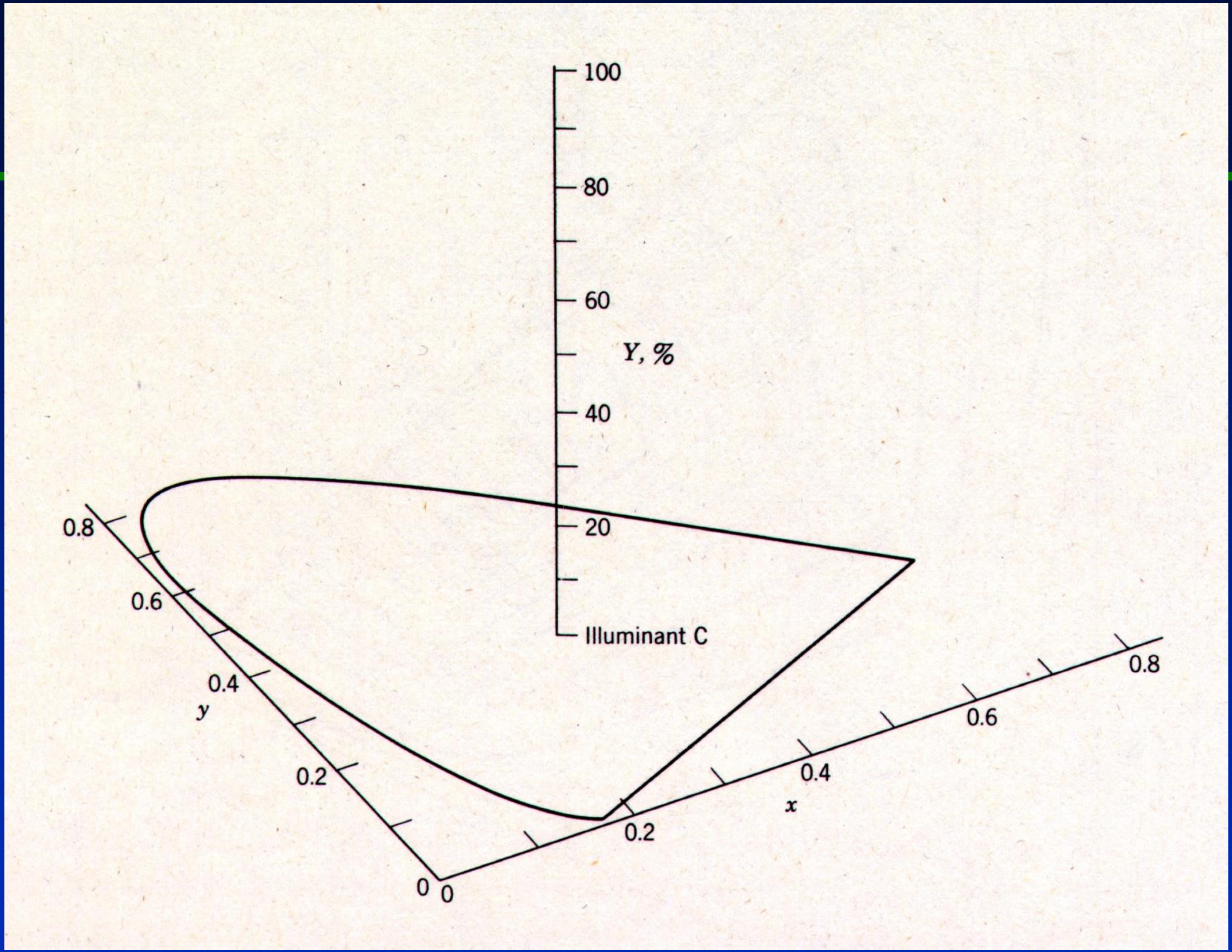




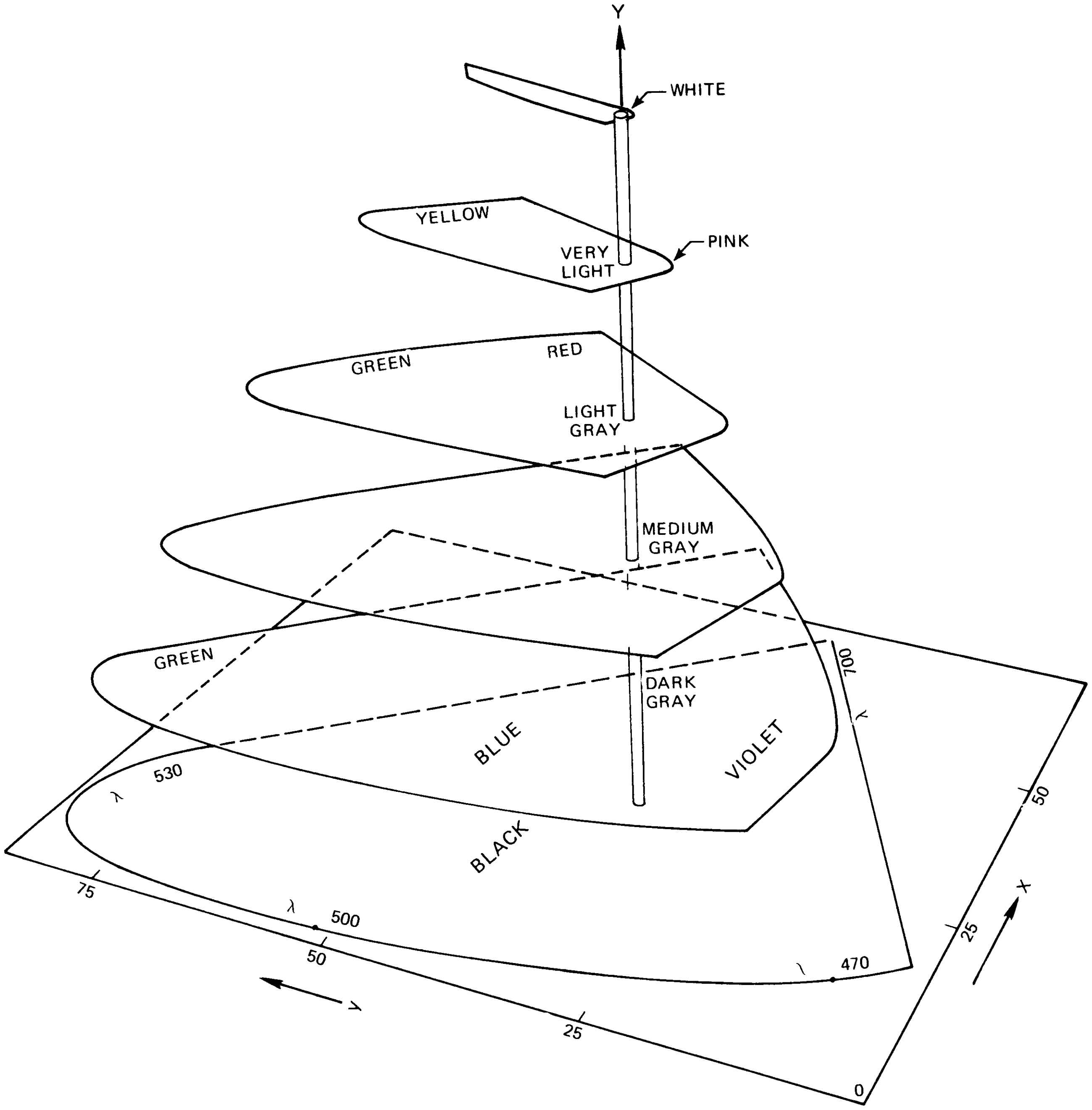
Chromaticity Diagram

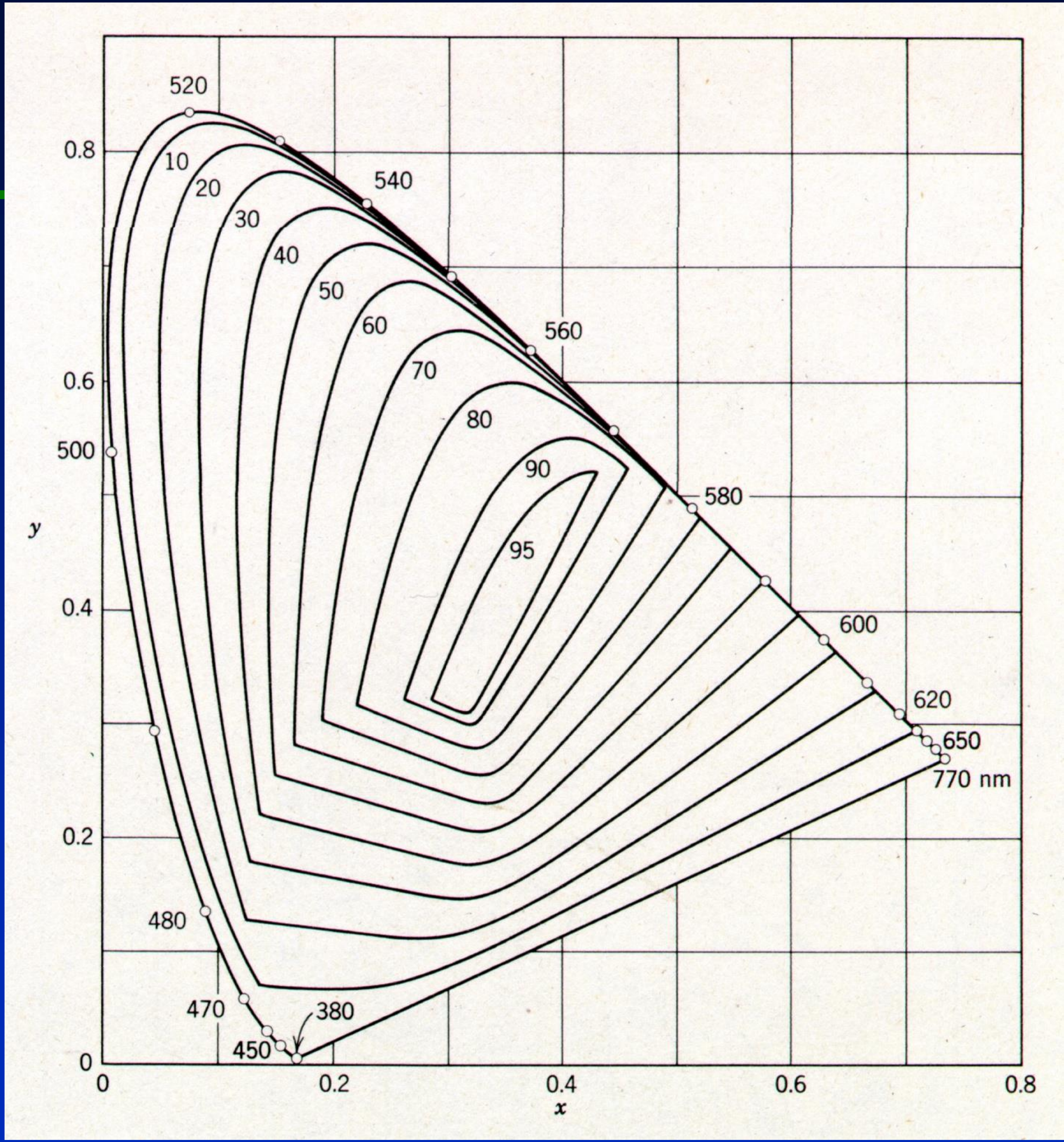






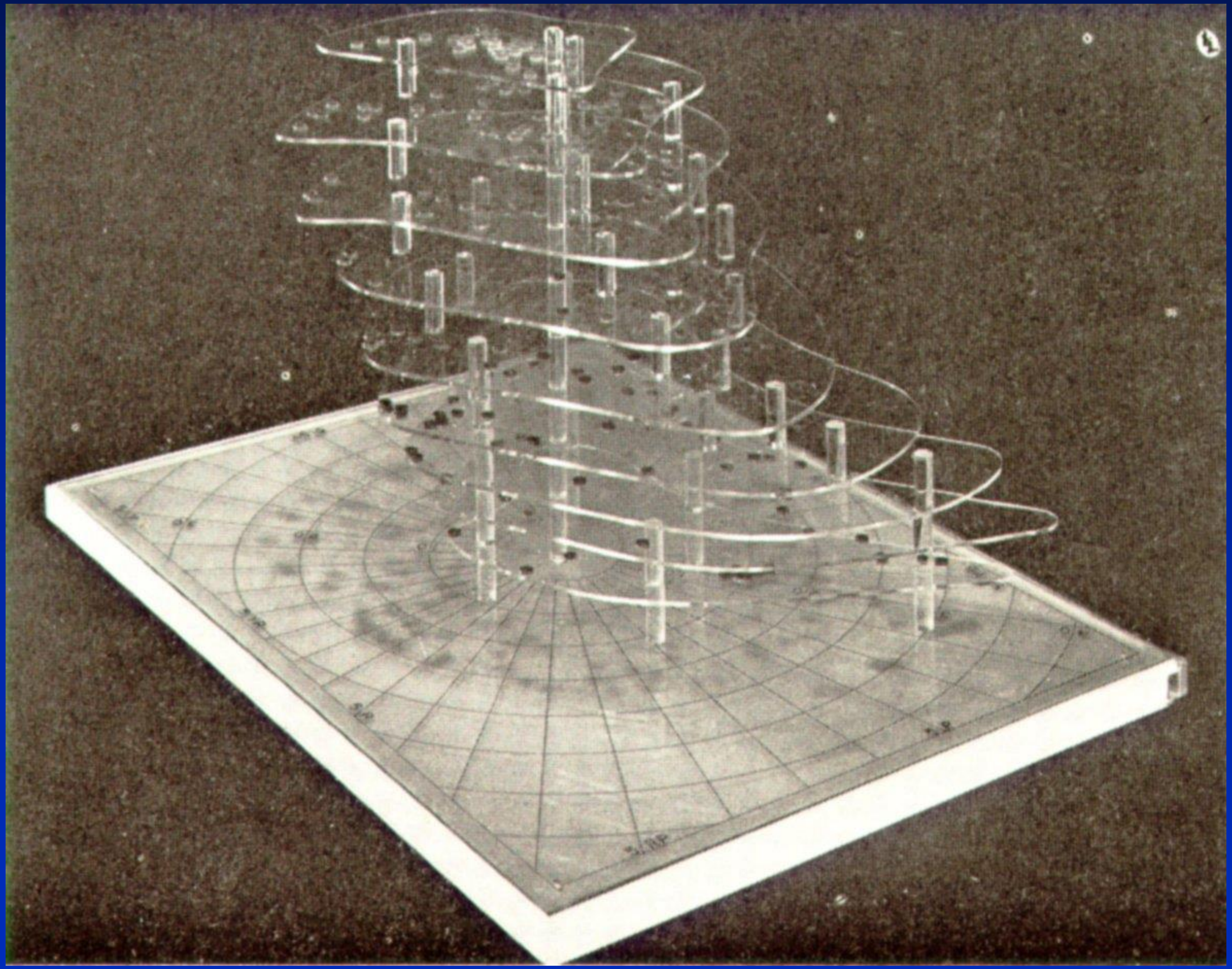
The luminance or lightness axis, when added to the chromaticity diagrams provides the third dimension for color.


Y
WHITE
YELLOW
VERY
LIGHT
PINK
GREEN
RED
LIGHT
GRAY
MEDIUM
GRAY
GREEN
DARK
GRAY
700
BLUE
VIOLET
530
BLACK
50
X
75
500
50
25
470
25
0



520
10
20
540
30
40
50
60
560
70
80
90
580
95
600
620
650
770 nm
500
480
470
450
380
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
y
x

3D Model – Billmeyer (Berns)



End. . .