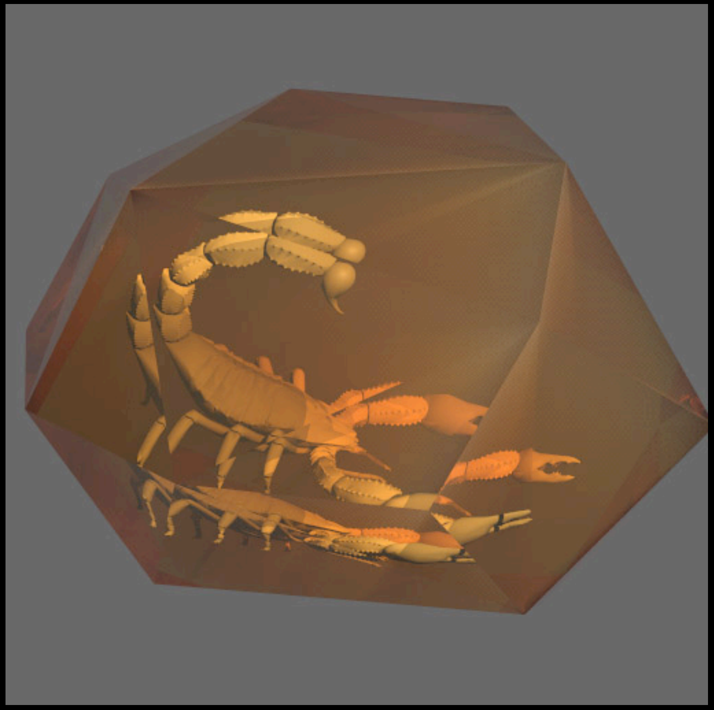
Single Scattering in Refractive Media with Triangle Mesh Boundaries



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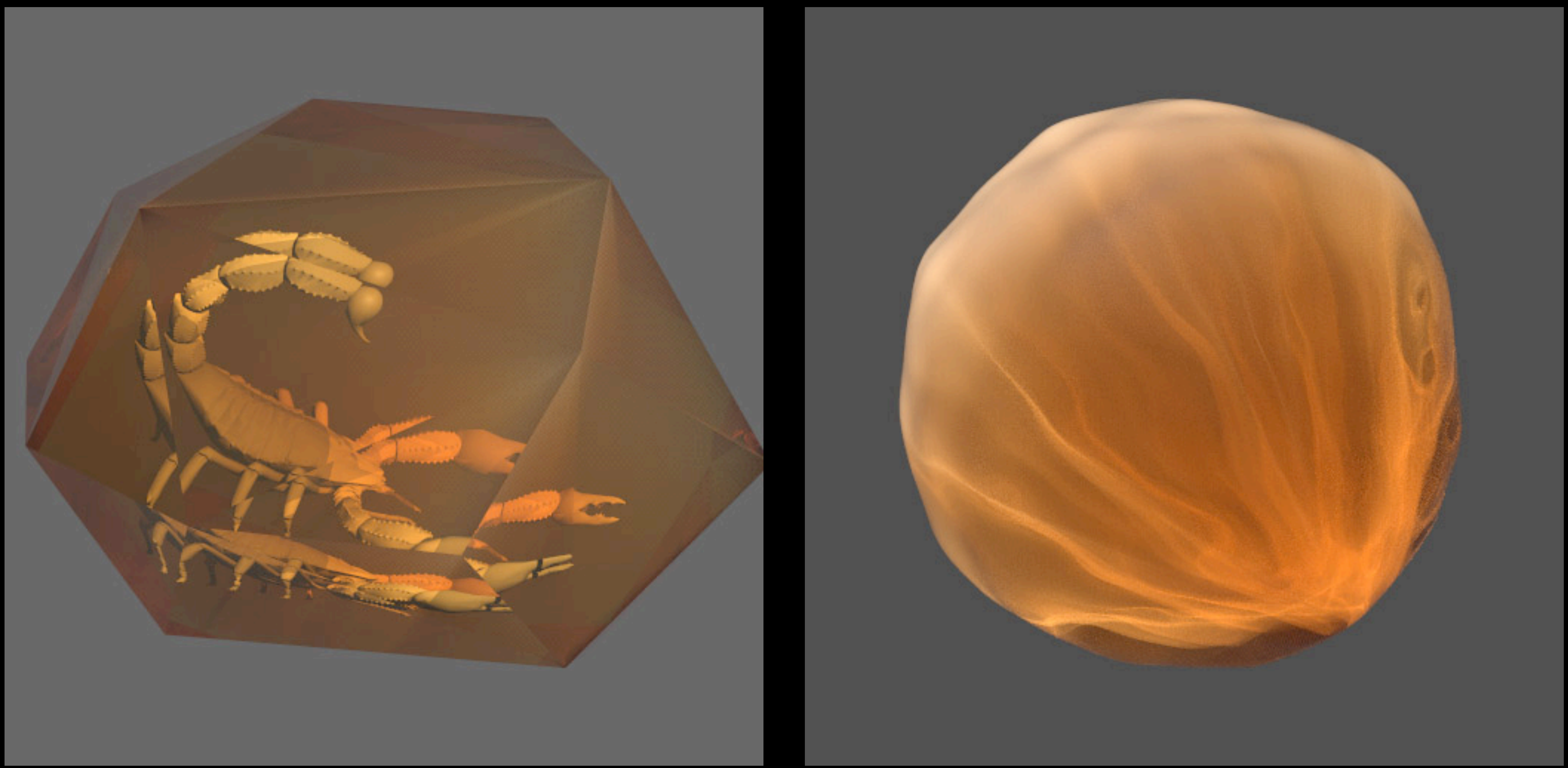
Presented at SIGGRAPH 2009

Single Scattering

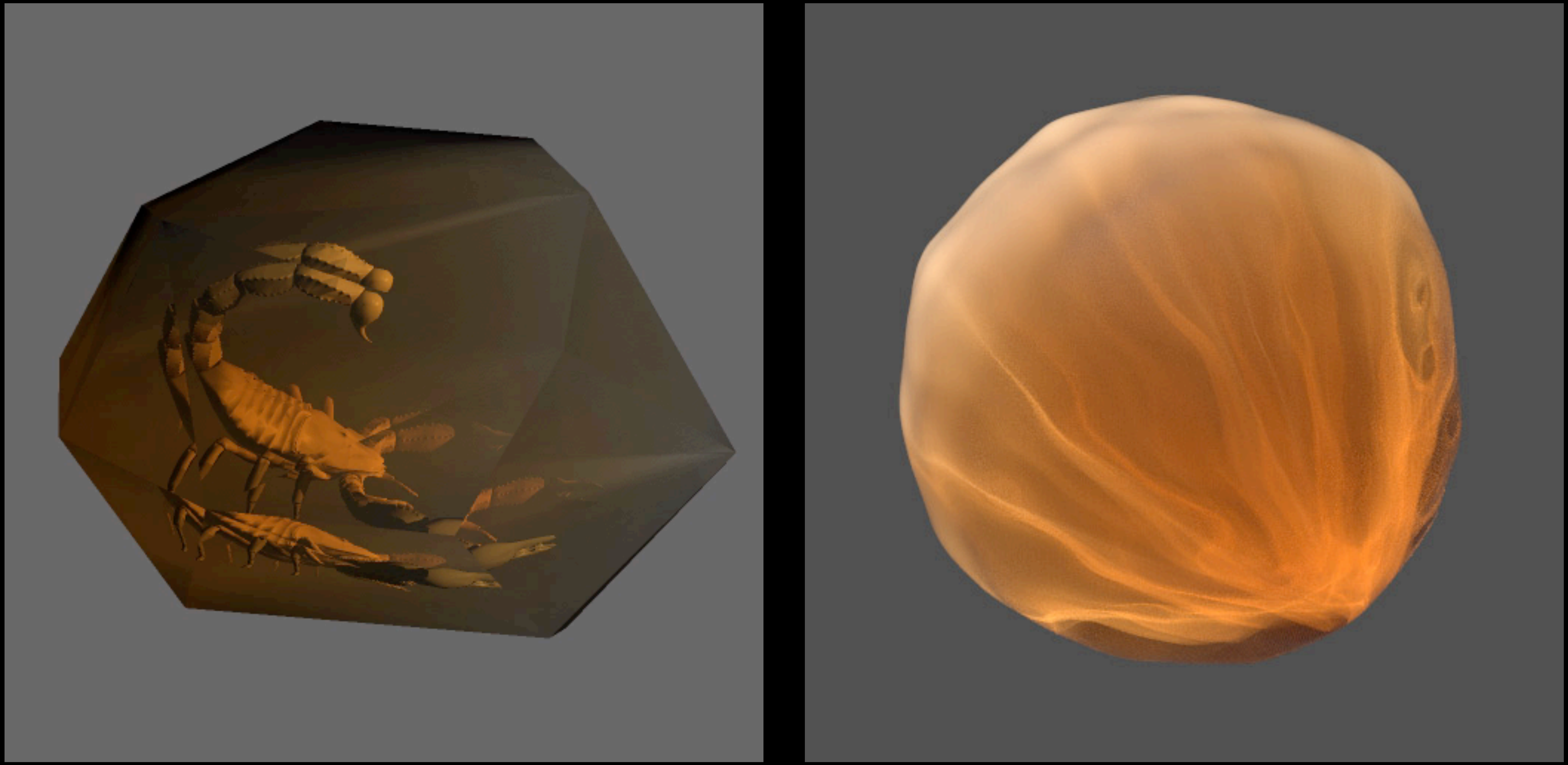
- Direct illumination in refractive objects is hard



Single scatter from a single point light source

Single Scattering

- Direct illumination in refractive objects is hard



Single scatter from a single point light source

Single Scattering

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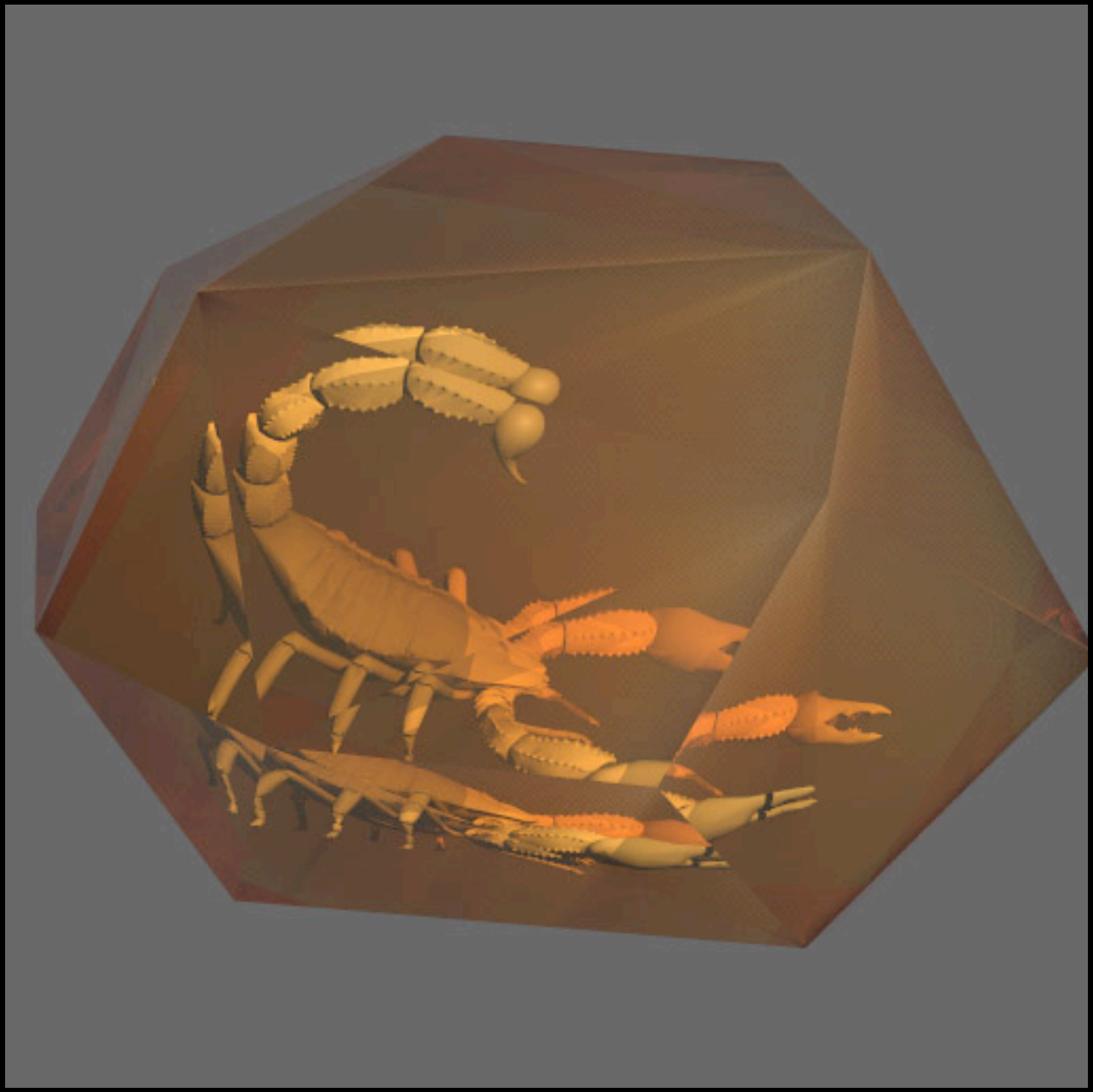
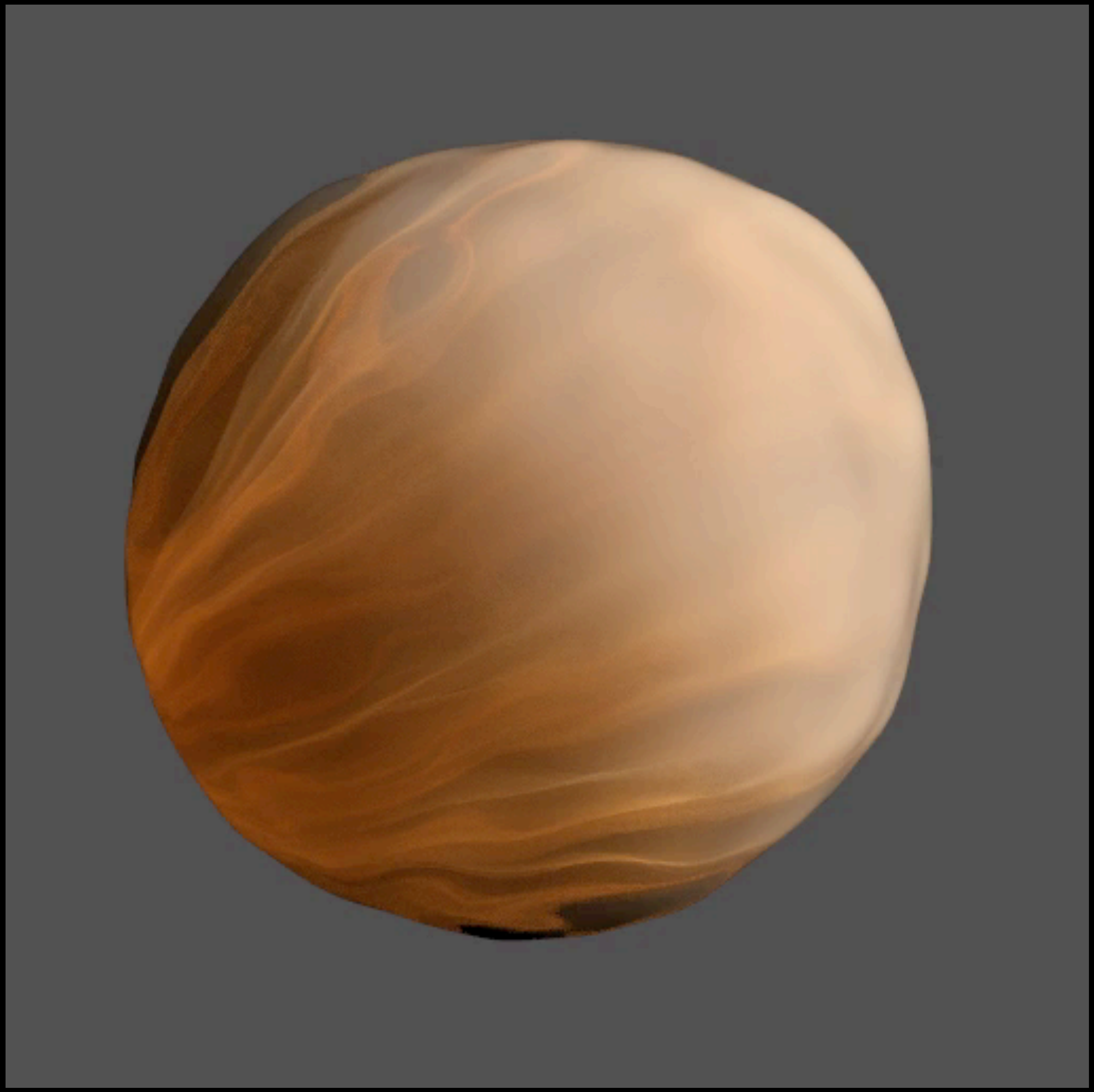


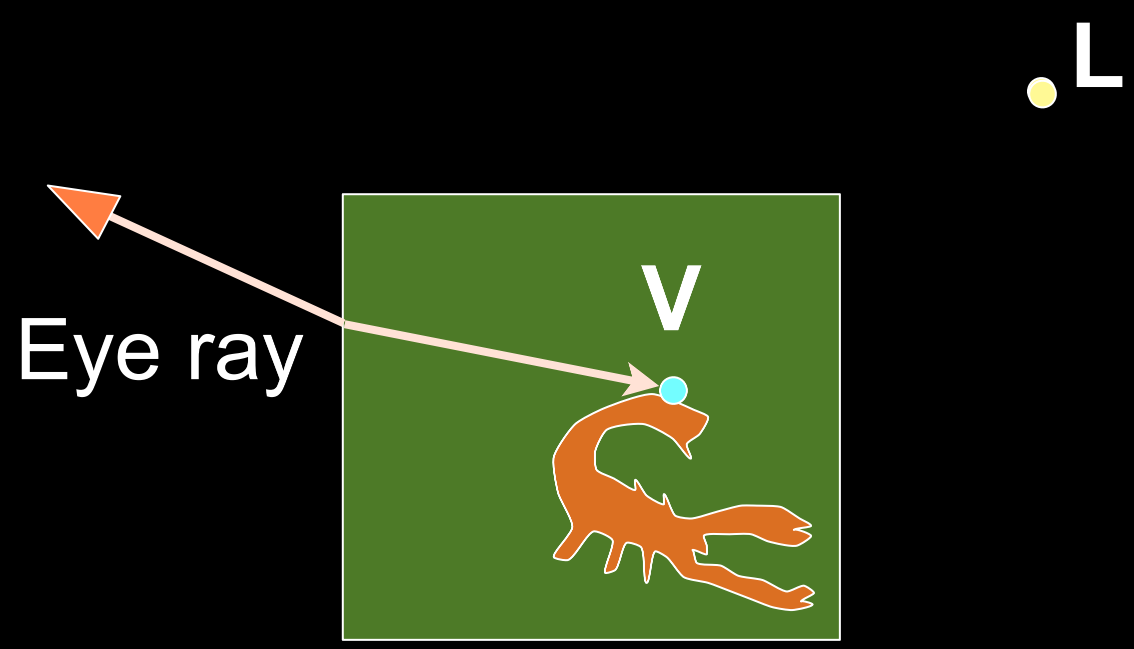


Single scatter from a single point light source

Problem

Problem

- Find direct illumination at V (receiver) from L (light)



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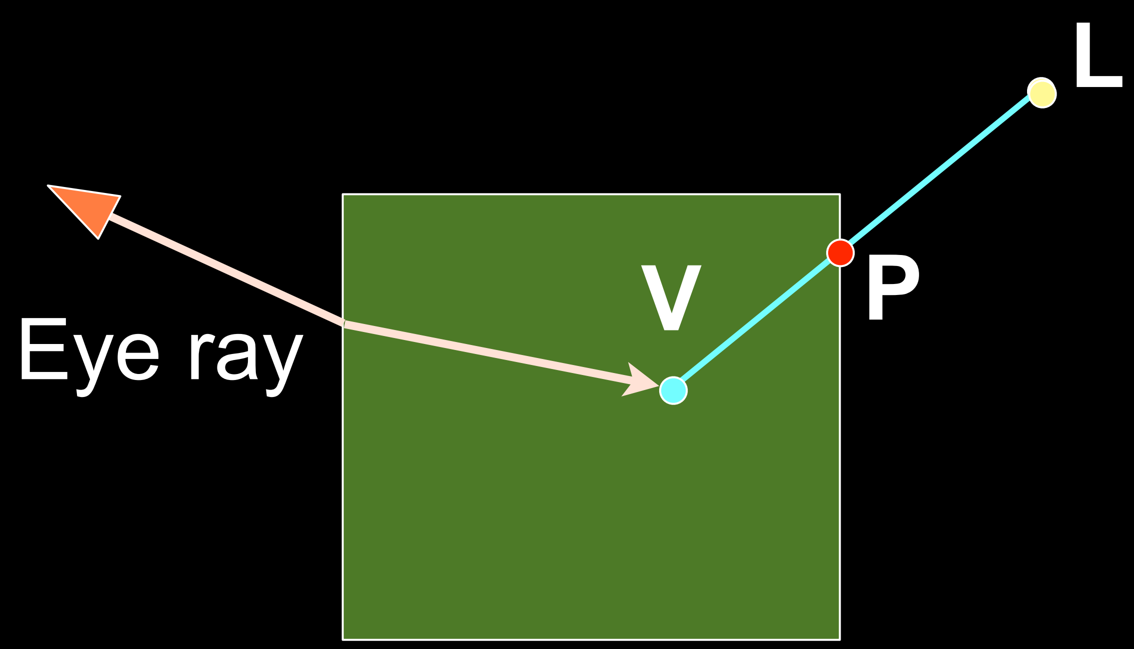
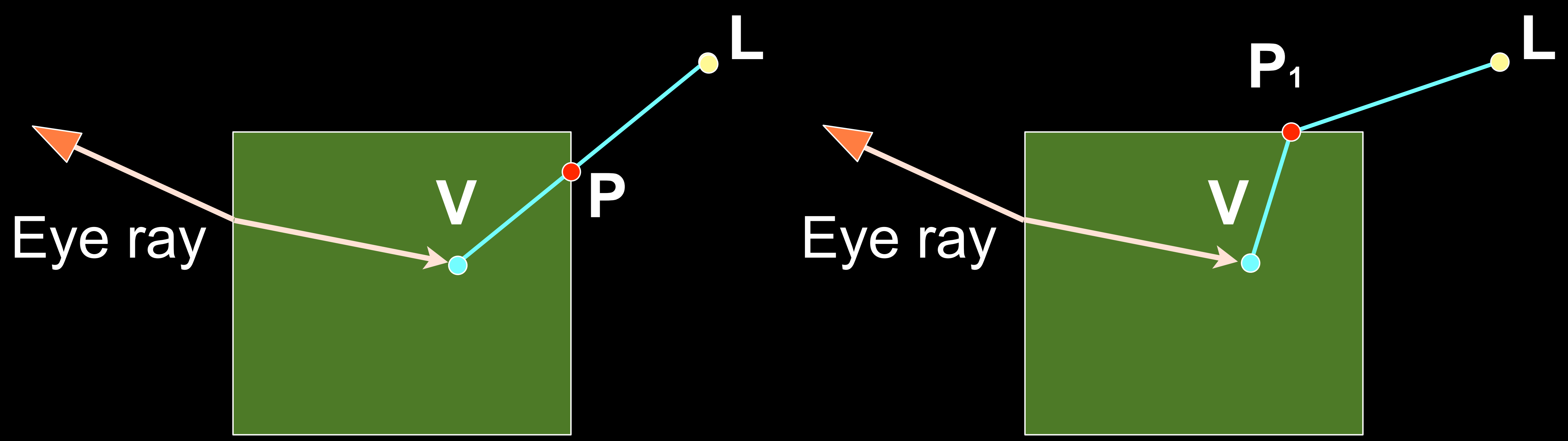


Ignoring refraction

Challenges: Bending of Path

- Find direct illumination at V (receiver) from L (light)
- Light bends at interface according to Snell's Law

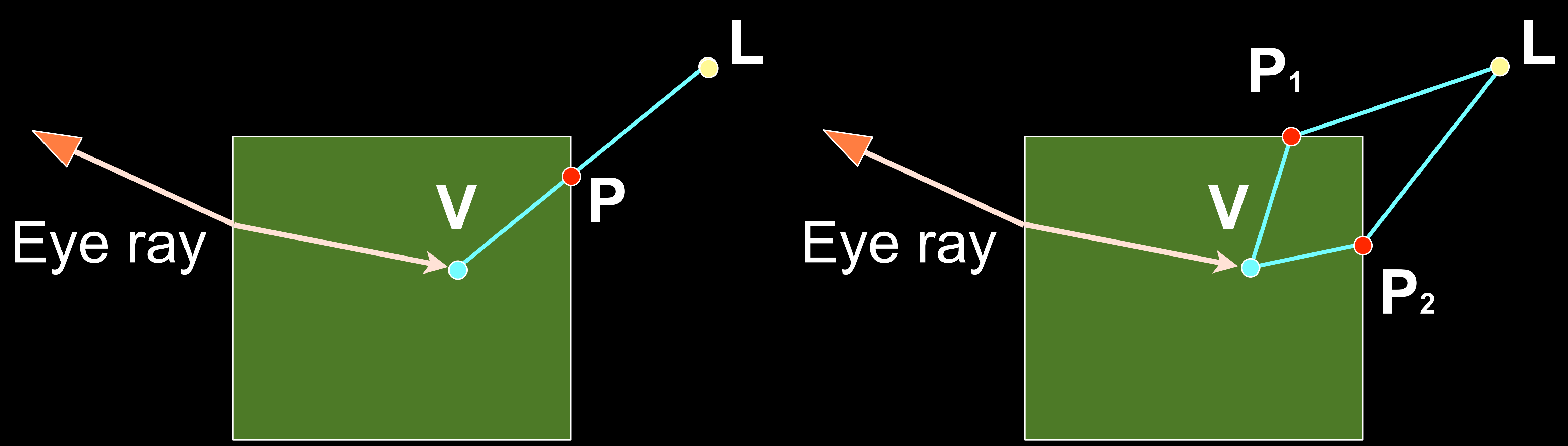


Ignoring refraction

With refraction

Challenges: Multiple Paths

- Find direct illumination at V (receiver) from L (light)
- Light bends at interface according to Snell's Law
  - Can have zero, one, or many such paths (and P's)

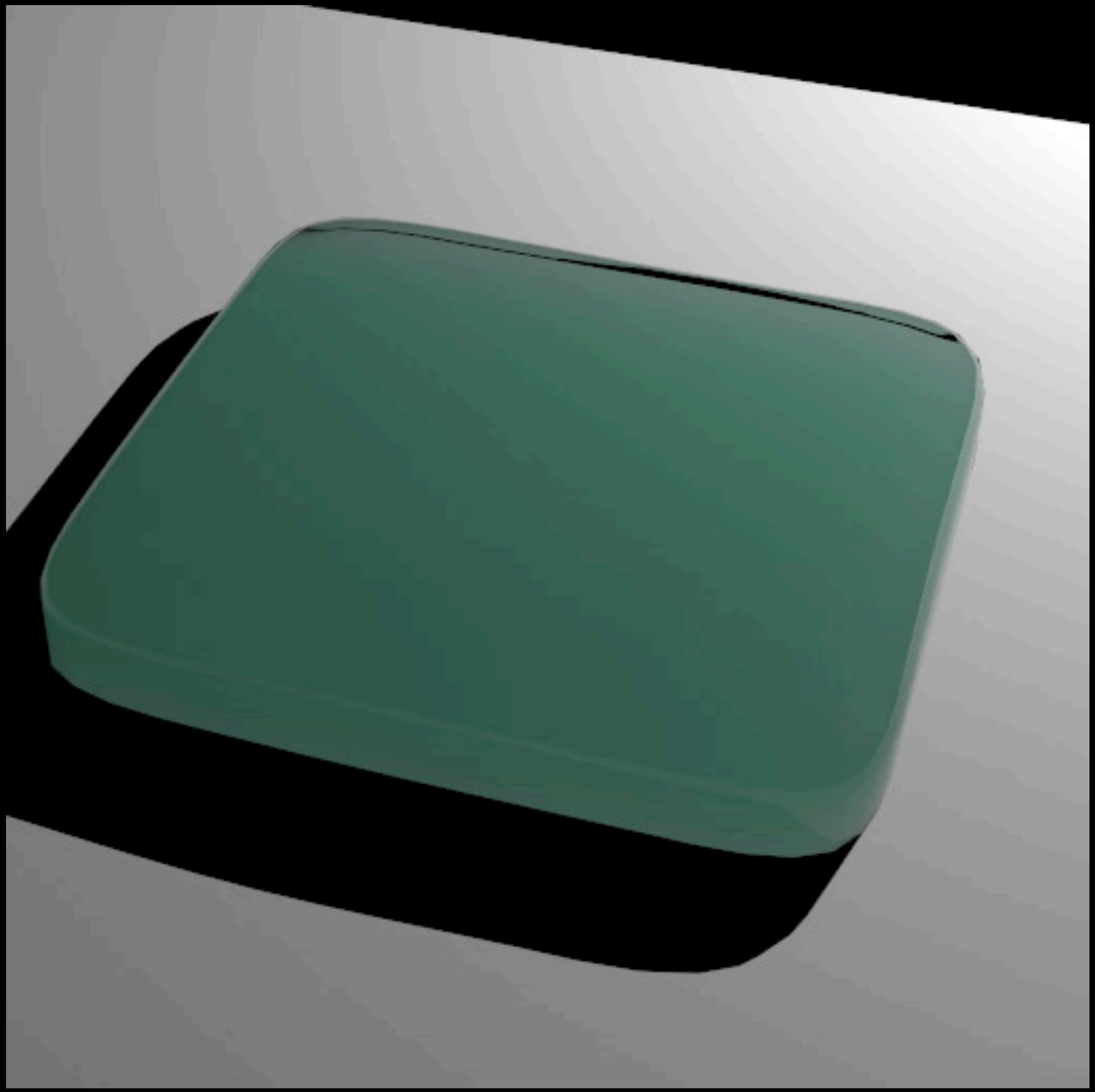


Ignoring refraction

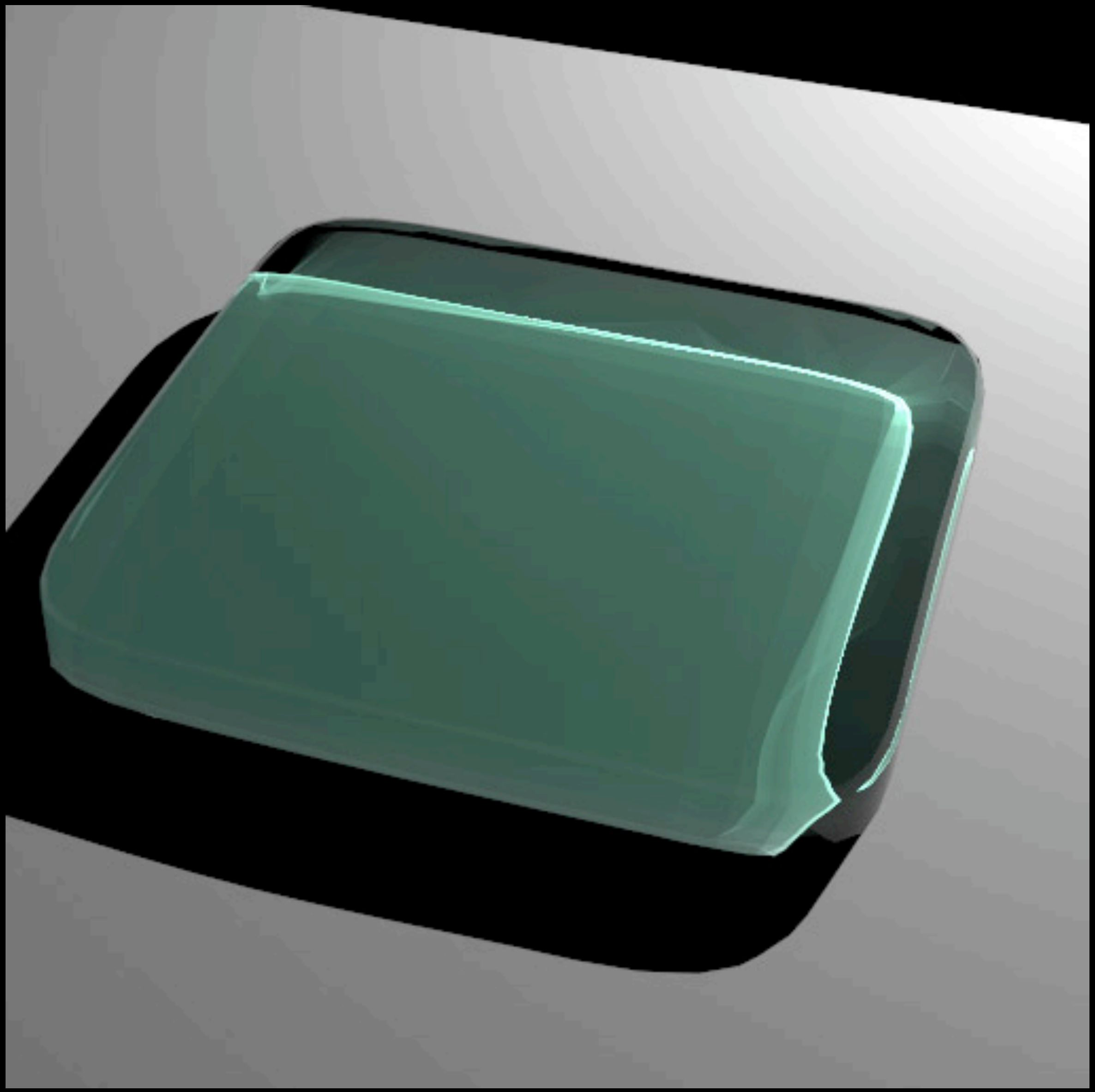
With refraction

Is it important?

- Glass tile quality comparison



Shadow rays ignore refraction



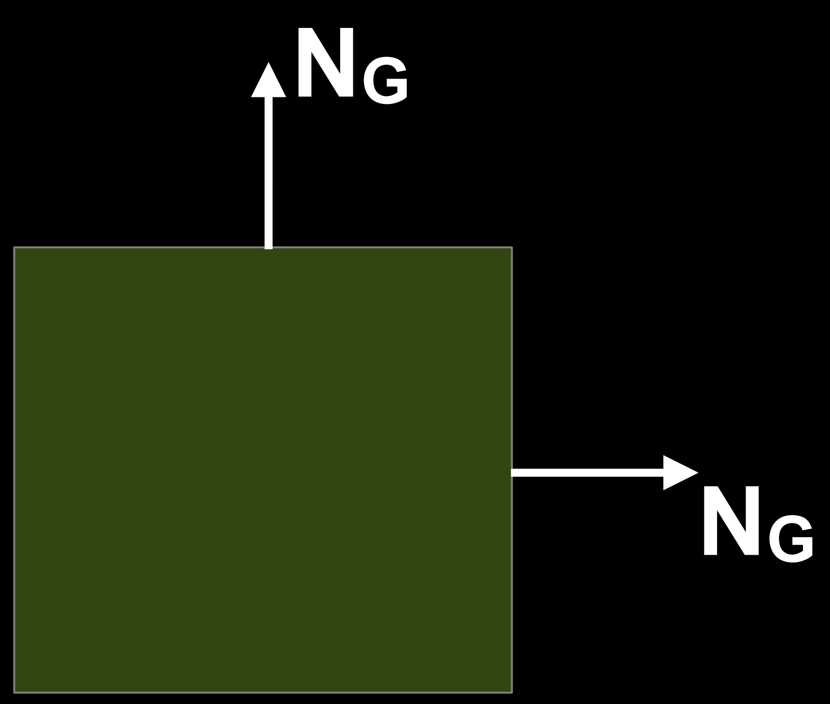
Our method

Challenges Summary

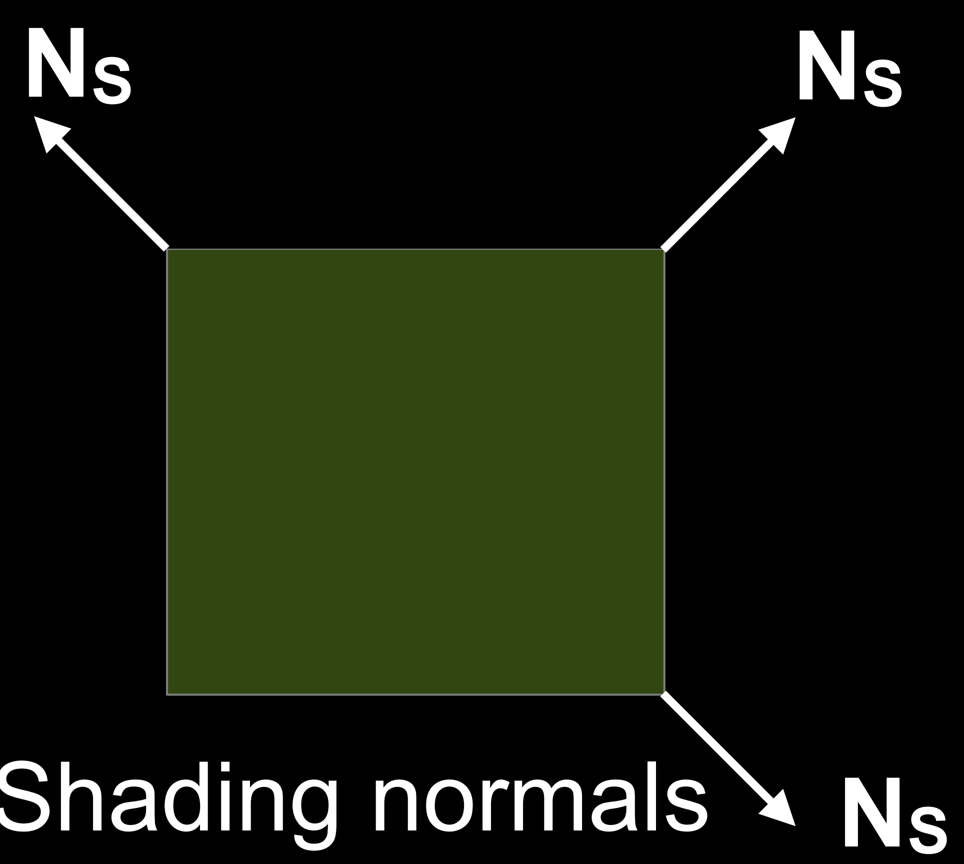
- Bending of paths
- Multiple paths
- Shading normals
- Large triangle meshes

Challenges: Shading Normals

- Geometric normals (N_G) vs. shading normals (N_S)
  - E.g., interpolated normals, bump maps, normal maps
  - Alters directions and intensities of light paths



Geometric normals



Shading normals

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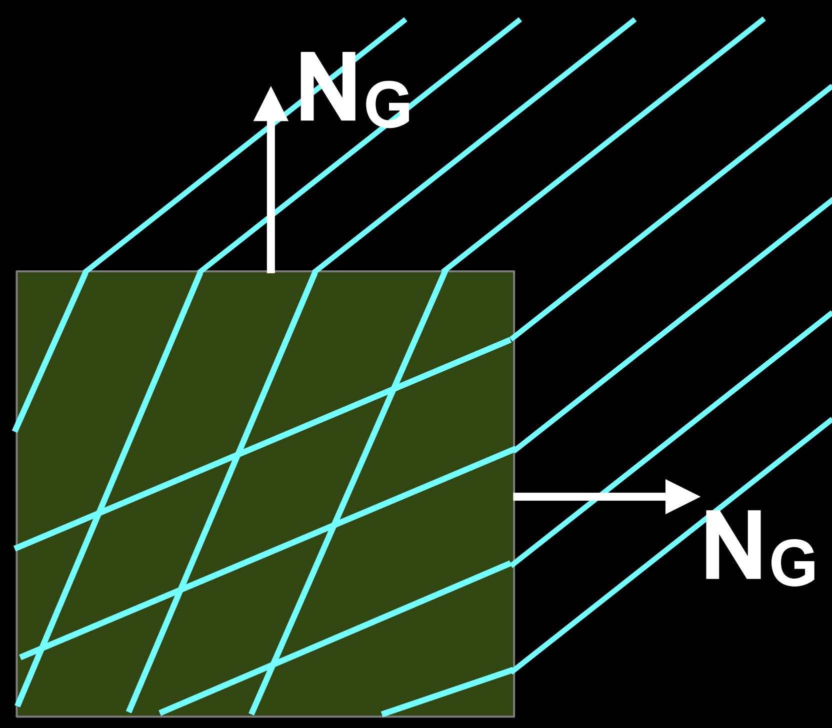
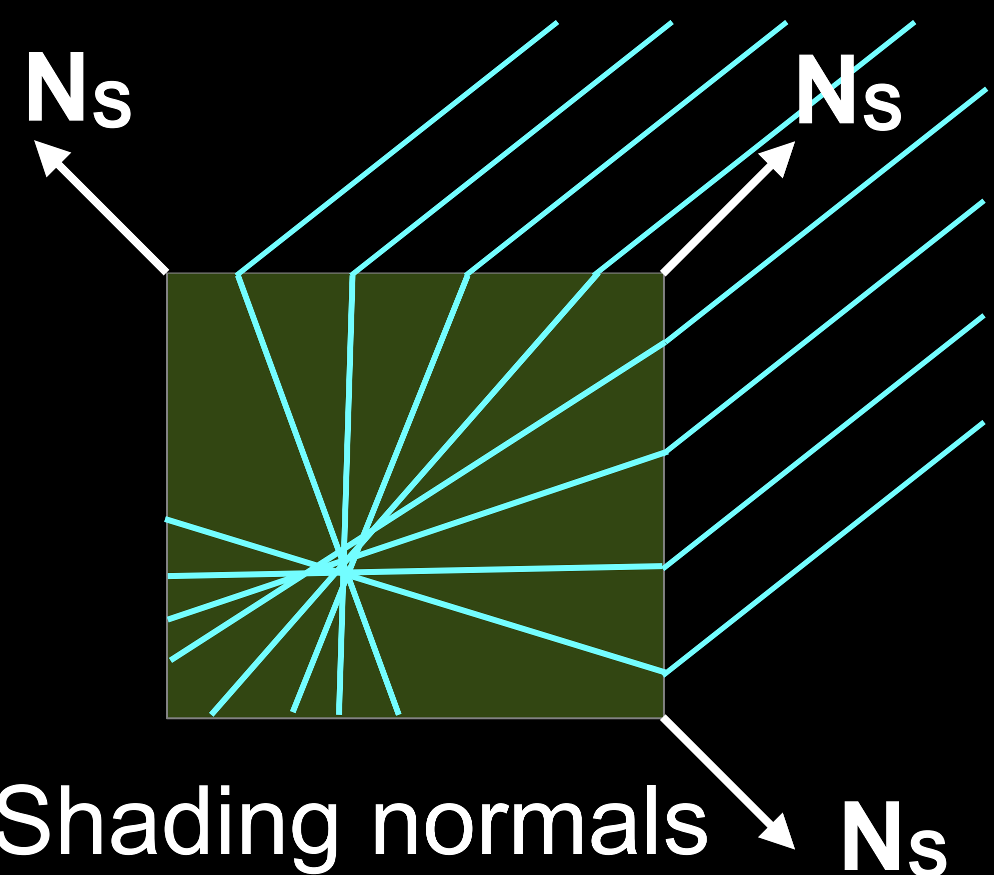


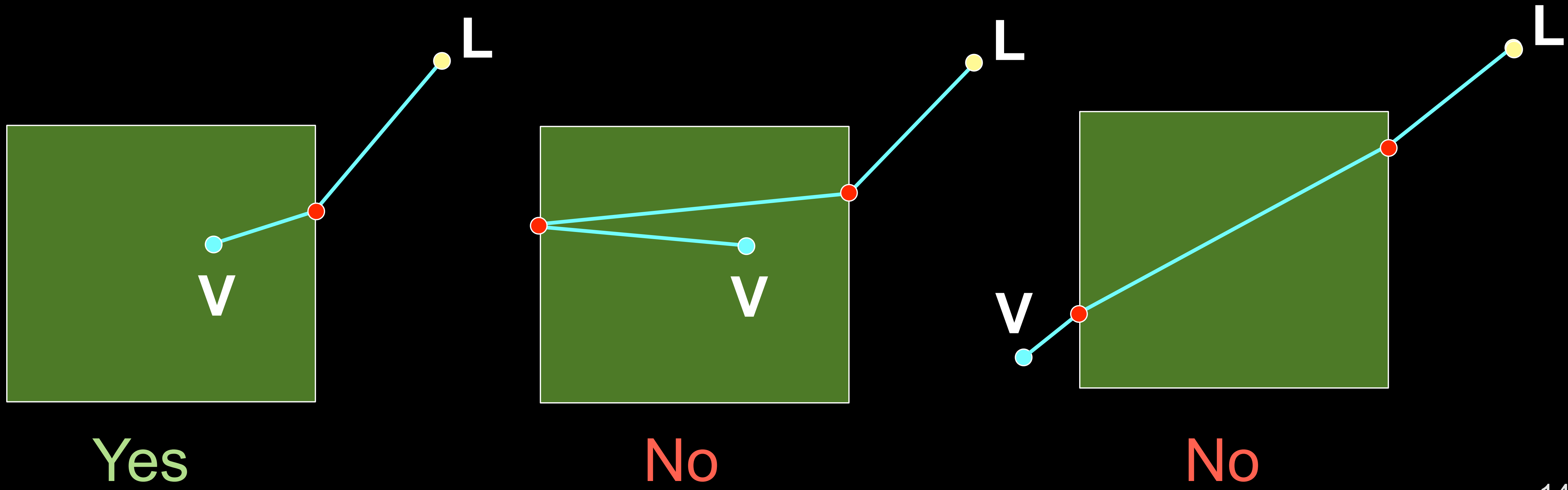
Geometric normals



Shading normals

Limitations

- Finds connections that
  - Cross the boundary exactly once
  - Have no other changes in direction
  - Cost depends on path count and boundary

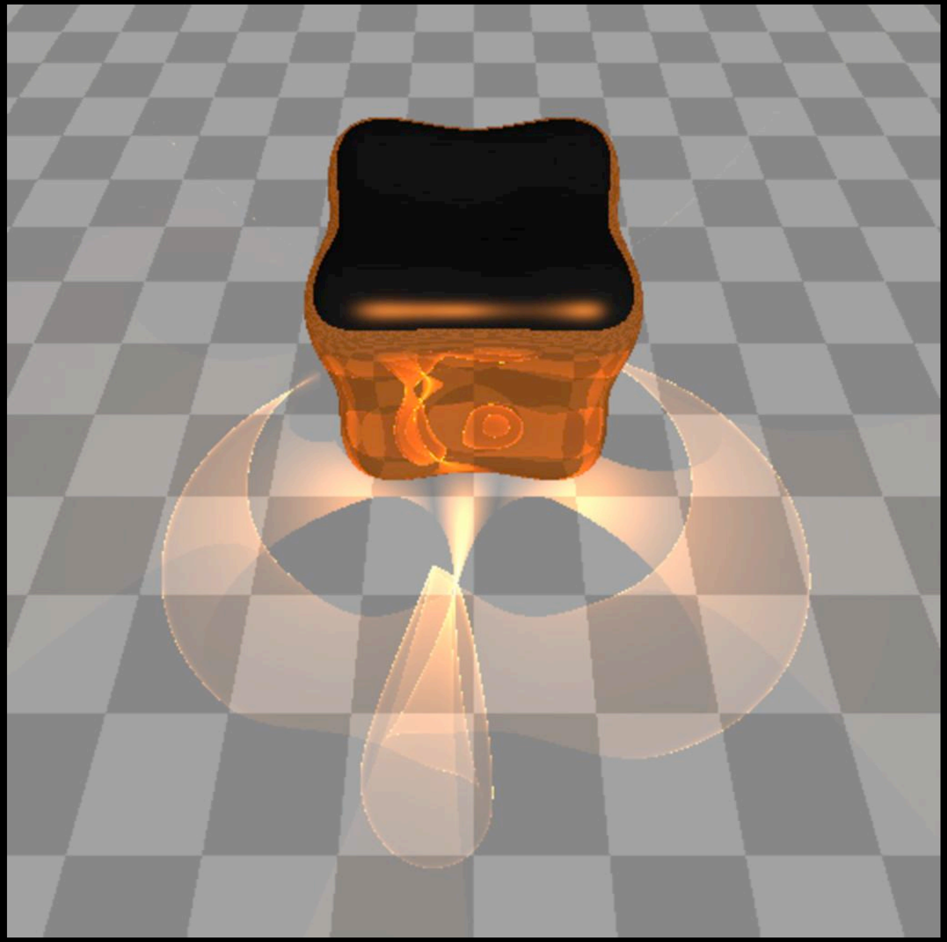


Prior Work

- Diffusion and multiple scatter
  - [eg, Stam 95, Jensen et al. 01, Wang et al. 08]
- Monte Carlo
  - [eg, Kajiya 86, Veach 97]
- Beam tracing
  - [eg, Nishita & Nakamae 94, Iwasaki et al. 03, Ernst et al. 05]
- Photon mapping
  - [eg, Jensen 01, Sun et al. 08, Jarosz et al. 08]
- Fermat's principle
  - [eg, Mitchell & Hanrahan 92, Chen & Arvo 00]

Prior Work

- Mitchell & Hanrahan 92
  - Used Fermat's principle and Newton's method
  - Reflection (shown) and refraction
- Limitations
  - Only supported implicit surfaces
  - Cannot handle shading normals
  - Expensive 3D search
  - Not scalable to complex geometry



Mitchell & Hanrahan 92

Contributions

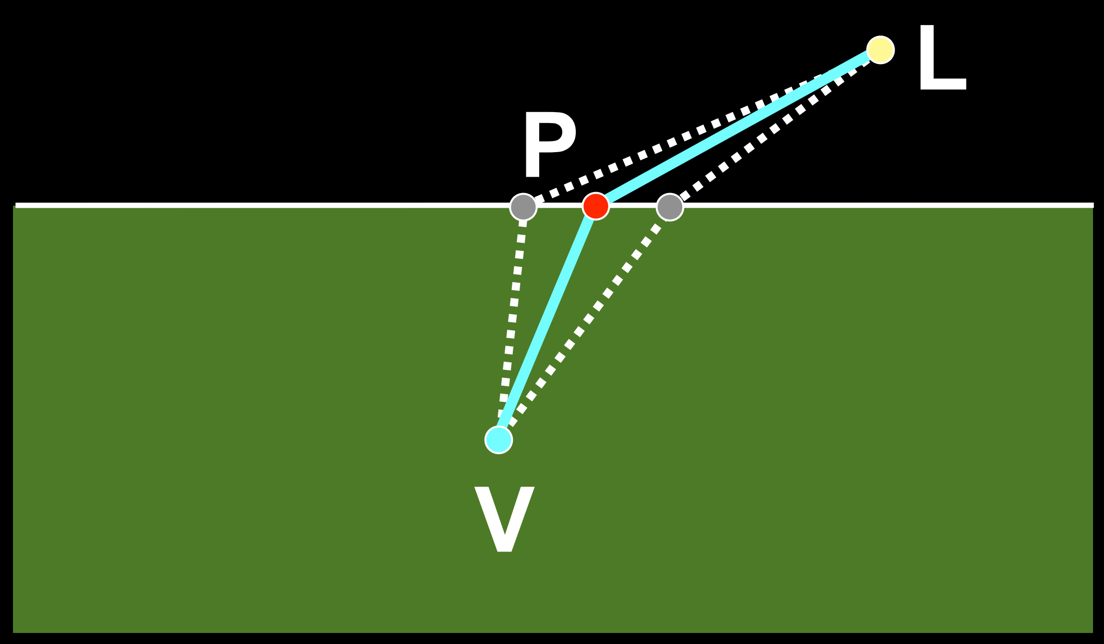
- Support triangles with shading normals
  - Most widely used geometry format
  - Required fundamental problem reformulation
  - New search methods and intensity equations
- Hierarchical culling
  - Scales to complex objects with many triangles
- CPU and GPU implementations
  - Interactive performance on some scenes

Outline

- Half-vector formulation
- Solving for a single triangle
- Hierarchical culling for meshes
- Results

Fermat's Principle

- Define optical path length
  - $\mathbf{d}(P) = \eta \, ||V-P|| + ||P-L||$
  - Extrema of $\mathbf{d}(P)$ are the refraction points
- Cannot handle shading normals

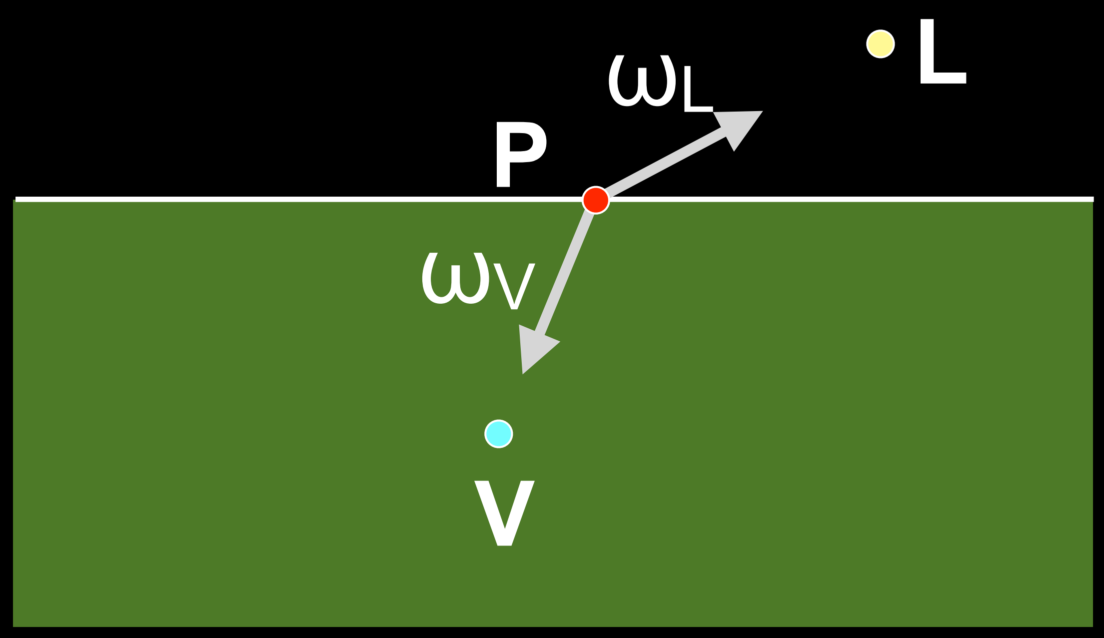


Index of refraction = η

Half-Vector Formulation

- Used in micro-facet model [Walter et al. 07]

- Direction to receiver: $\omega_V = (V - P) / ||V - P||$
- Direction to light: $\omega_L = (L - P) / ||L - P||$



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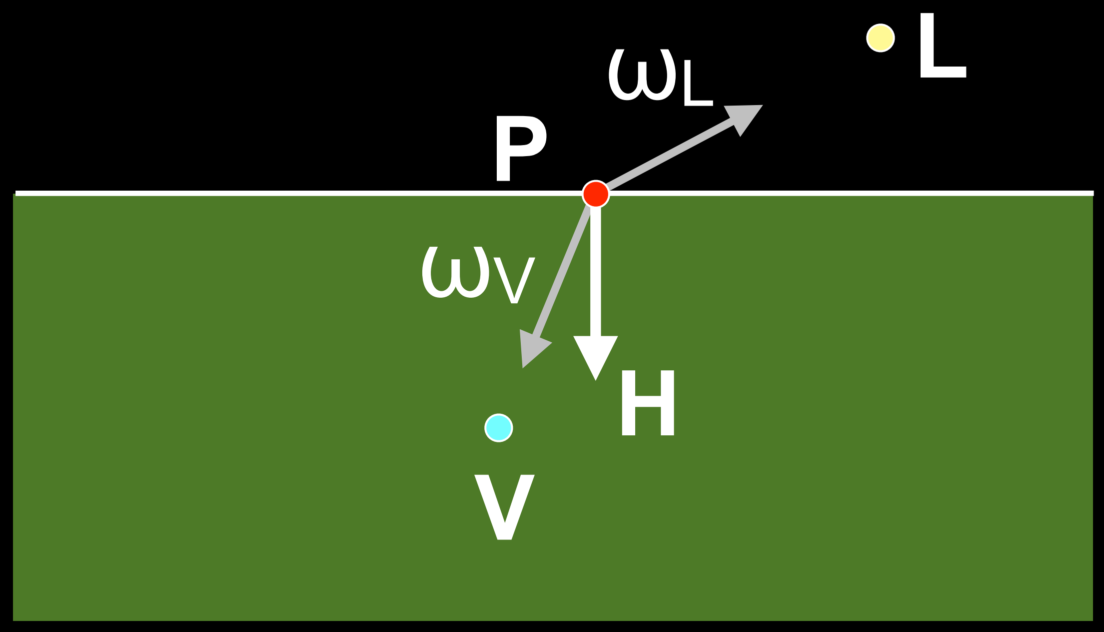
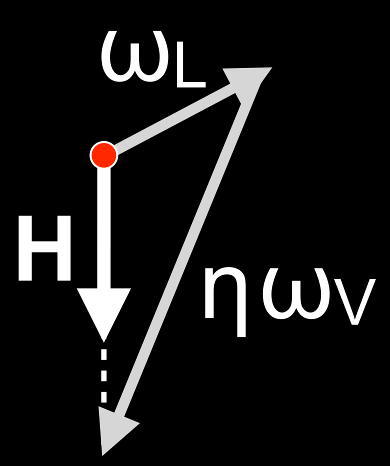
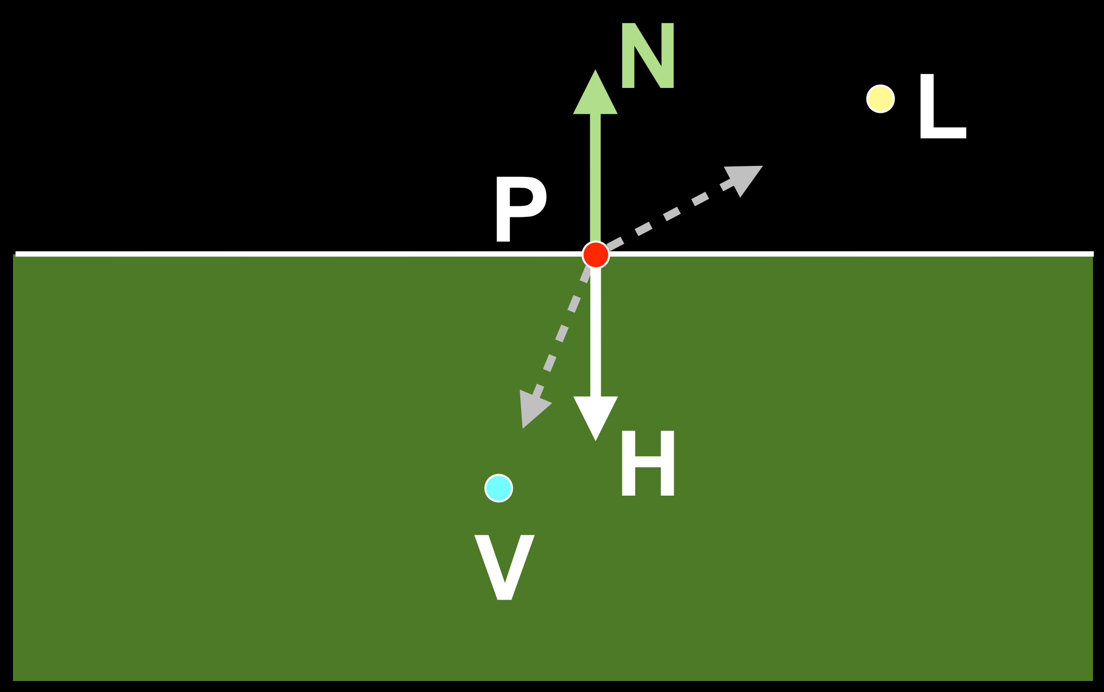
- Direction to receiver: $\omega_V = (V - P) / ||V - P||$
- Direction to light: $\omega_L = (L - P) / ||L - P||$
- Half-vector: $H = (\eta\,\omega_V + \omega_L) / ||\eta\,\omega_V + \omega_L||$





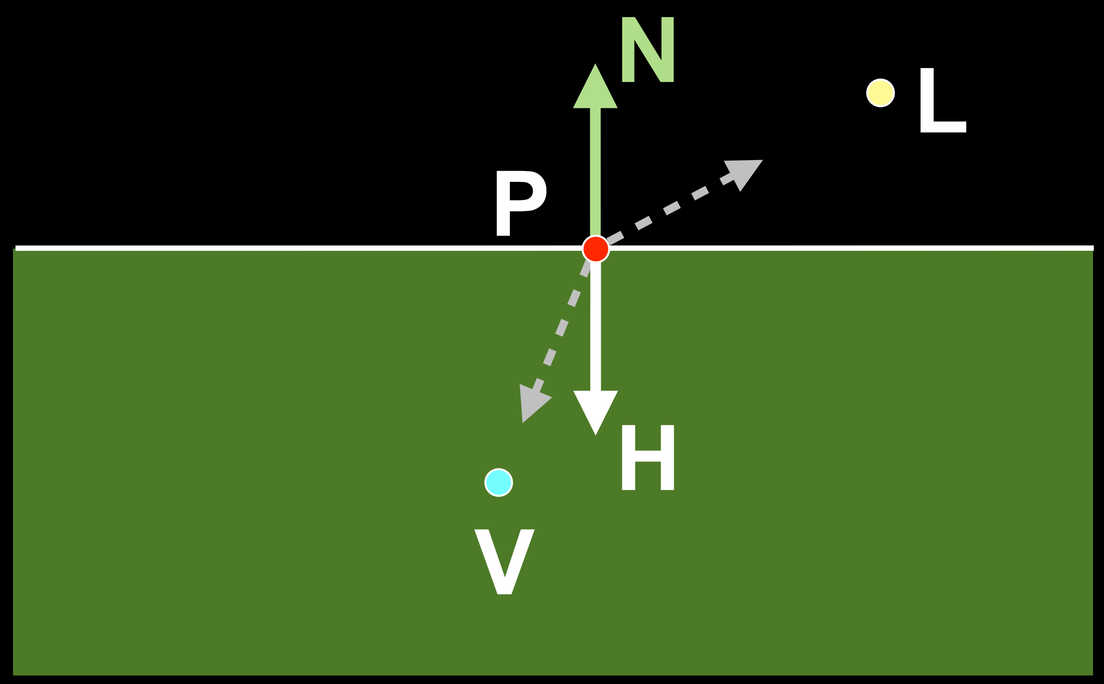
Half-Vector Formulation

- If H = -N (surface normal) then
  - V, P, L, and N are coplanar
  - Angles obey Snell's Law: $\eta \sin(\theta_V) = \sin(\theta_L)$
- It is a refraction solution
  - Assuming V and L lie on the correct sides of the normal



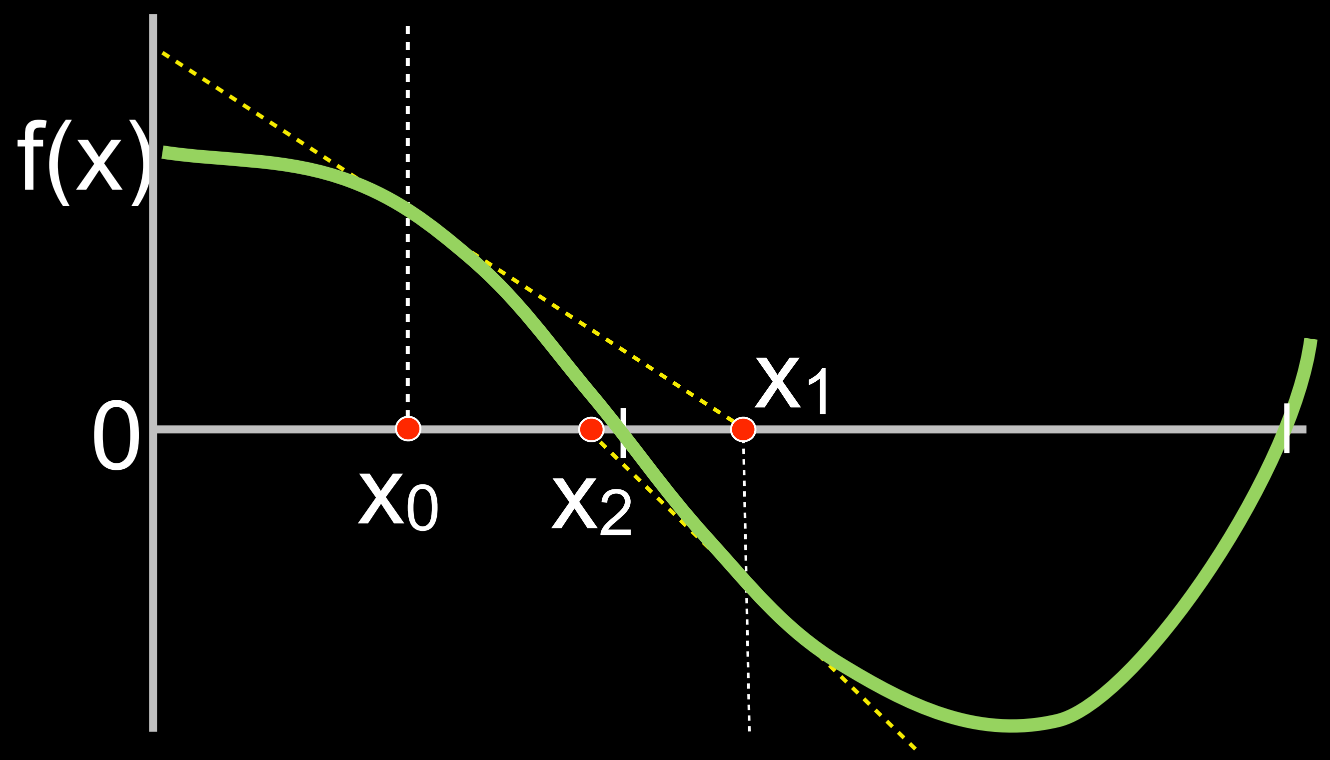
Half-Vector Formulation

- Find all P such that: H + N = 0
  - Natural extension to shading normals: $H + N_s = 0$
- Newton's method to find zeroes of: $\mathbf{f}(P) = H + N$



Newton's Method Review

- Quadratically convergent near a root
  - Each iteration doubles the precision
- Chaotic behavior far from a root
  - May diverge or converge to other roots

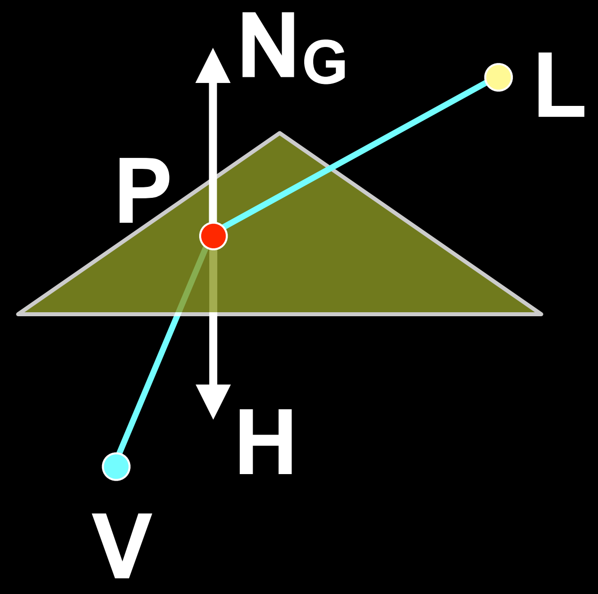


Outline

- Half-vector formulation
- Solving for a single triangle
  - Geometric normal - 1D Newton
  - Shading normals - 2D Newton
  - Subdivision oracles
- Hierarchical culling for meshes
- Results

Triangle Without Shading Normals

- Solution must lie in plane containing V, L, and N_G
  - Unique solution always exists
  - Simple 1D Newton's method converges
    - Typically in just 2 to 4 iterations
  - Check if solution lies within the triangle



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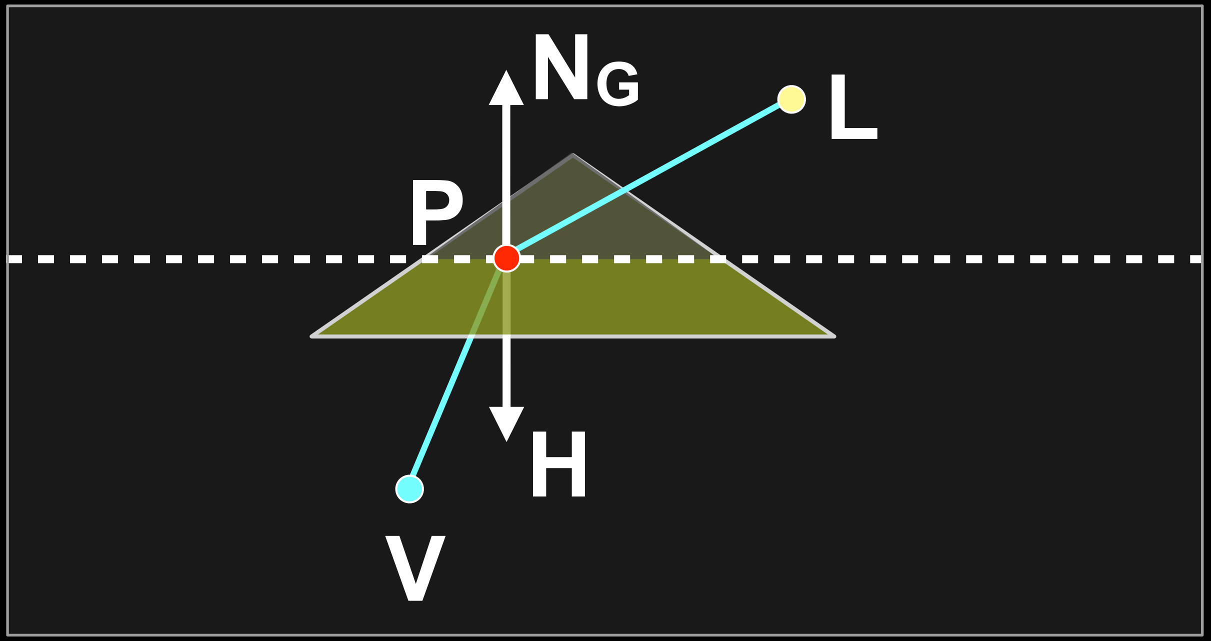


Triangle with Shading Normal

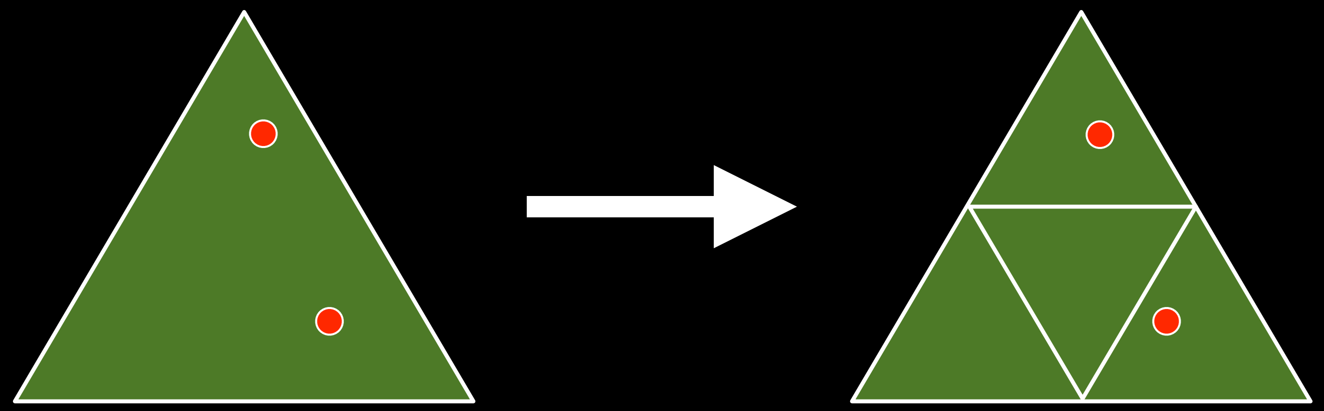
- Shading normal, N_S, varies over triangle
  - Full 2D search over triangle's area
- Function **f**(P) = H + N_S maps 2D to 3D
  - Derivative is 2X3 Jacobian matrix is non-invertible
  - Use pseudoinverse for Newton's method:

$$J^+ = (J^T J)^{-1} J^T$$

$$X_{i+1} = X_i - J^+(X_i)\mathbf{f}(X_i)$$

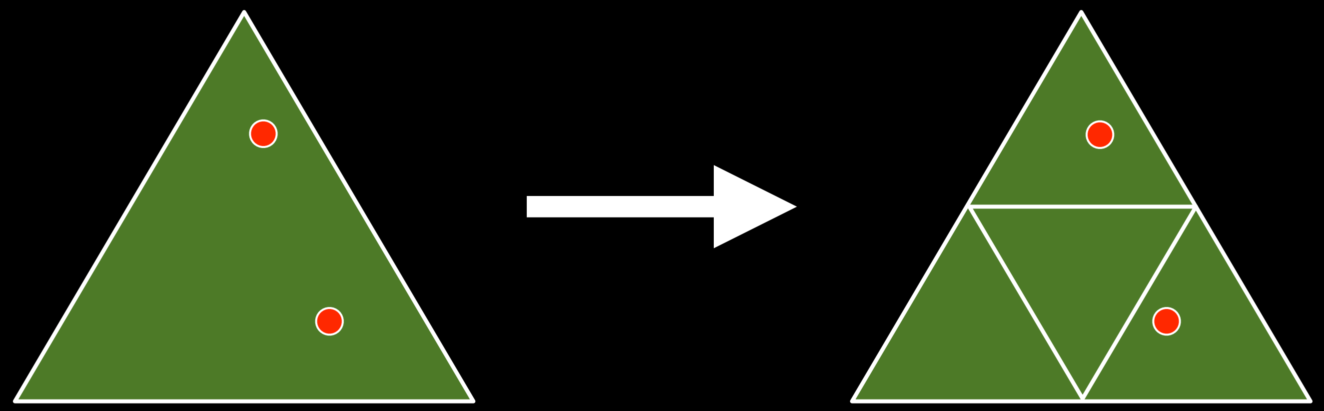
Triangle with Shading Normal

- Need good starting points
- May have zero, one, or multiple solutions
  - Subdivide triangle as needed to isolate solutions



Two Triangle Subdivision Oracles

- Test with strong guarantees
  - Based on [Krawczyk 69], [Mitchell&Hanrahan 92]
  - Conditions guarantee uniqueness and convergence
- Fast empirical heuristic
  - Based on solid angles of triangle and normals



Triangle summary

- For each triangle:
  - If no shading normals
    - Solve for P with 1D Newton
  - Else if passes the subdivision oracle
    - Solve for P with 2D Newton
  - Else
    - Subdivide into 4 triangles and try again
  - Test if P lies within the triangle

Outline

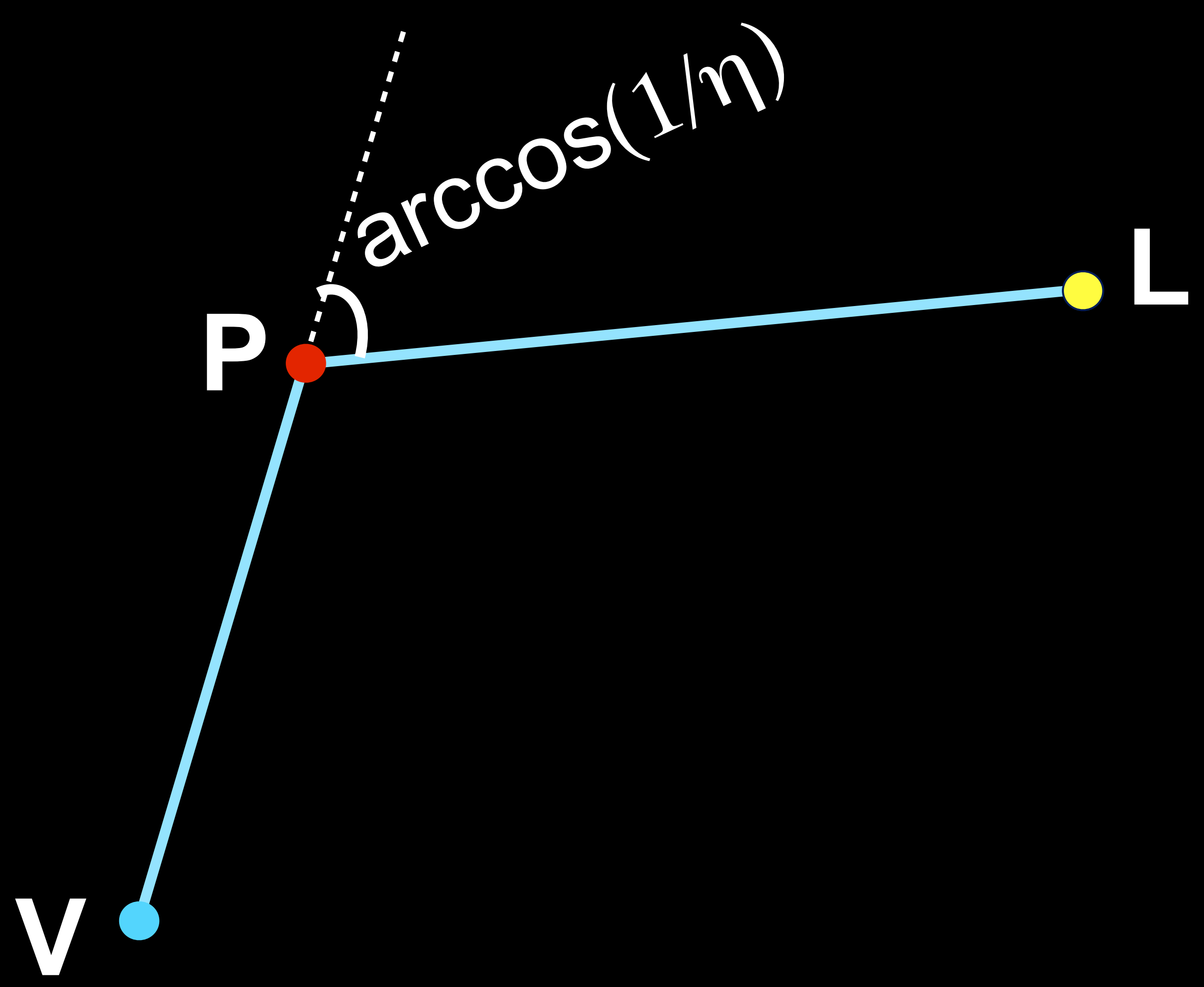
- Half-vector formulation
- Solving for a single triangle
- Hierarchical culling for meshes
- Results

Culling Tests

- Most triangles contain no solutions for P
- Three quick culling tests
  - Spindle
  - Sidedness
  - Interval

Spindle Culling Test

- Refraction bends path by angle ≤ arccos($1/\eta$)
  - Solutions must lie within circular arc (2d) or spindle (3d)



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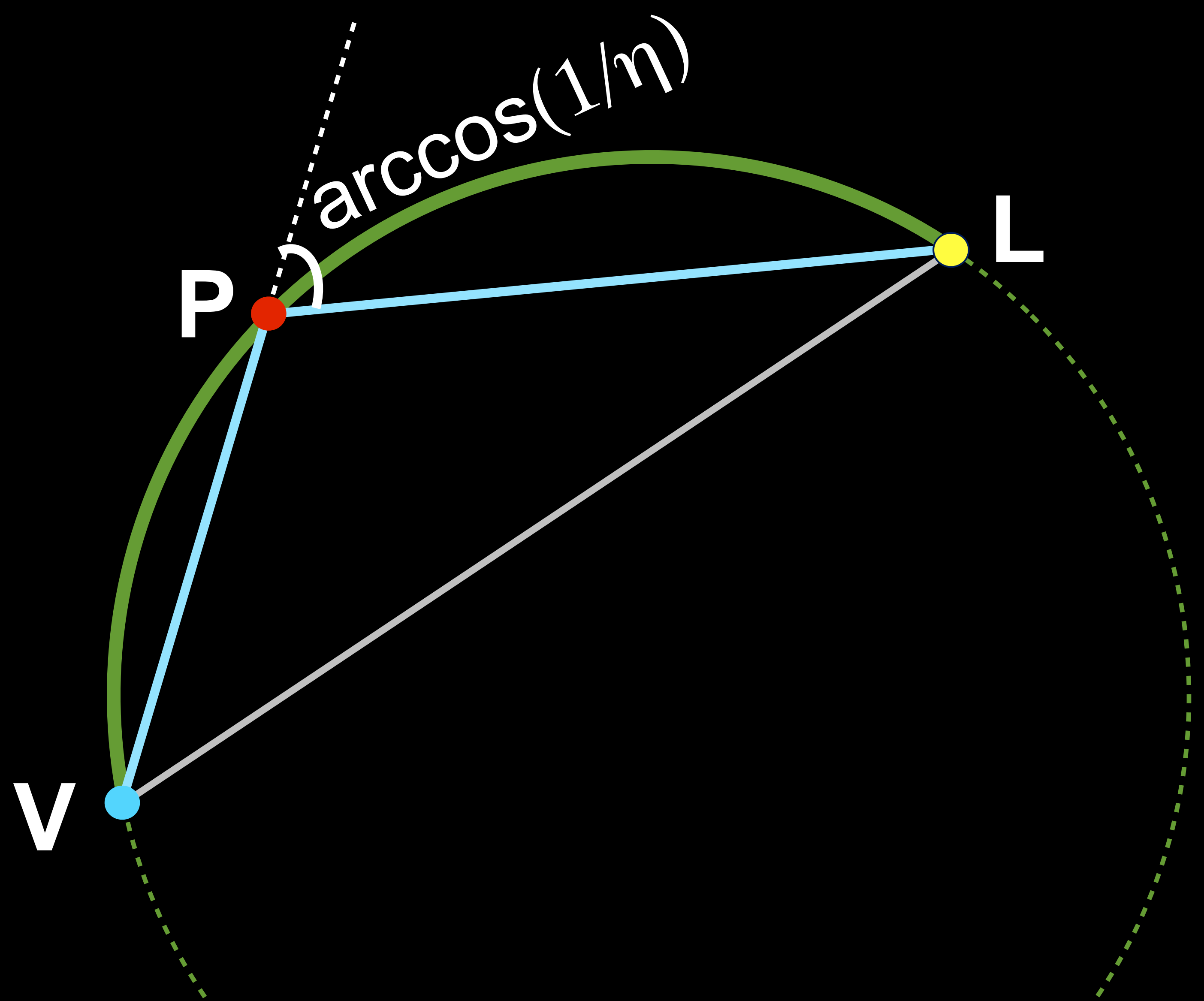

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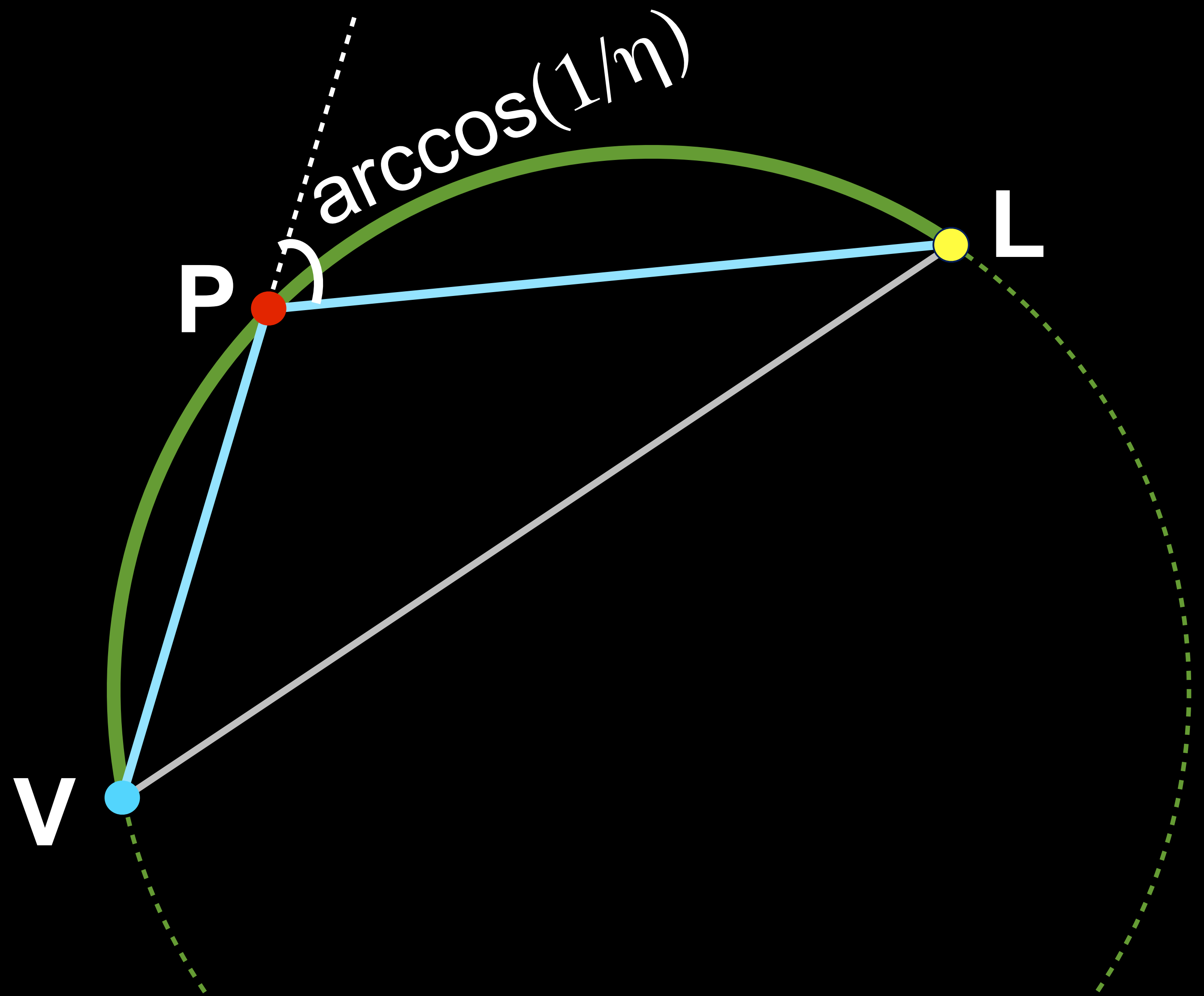
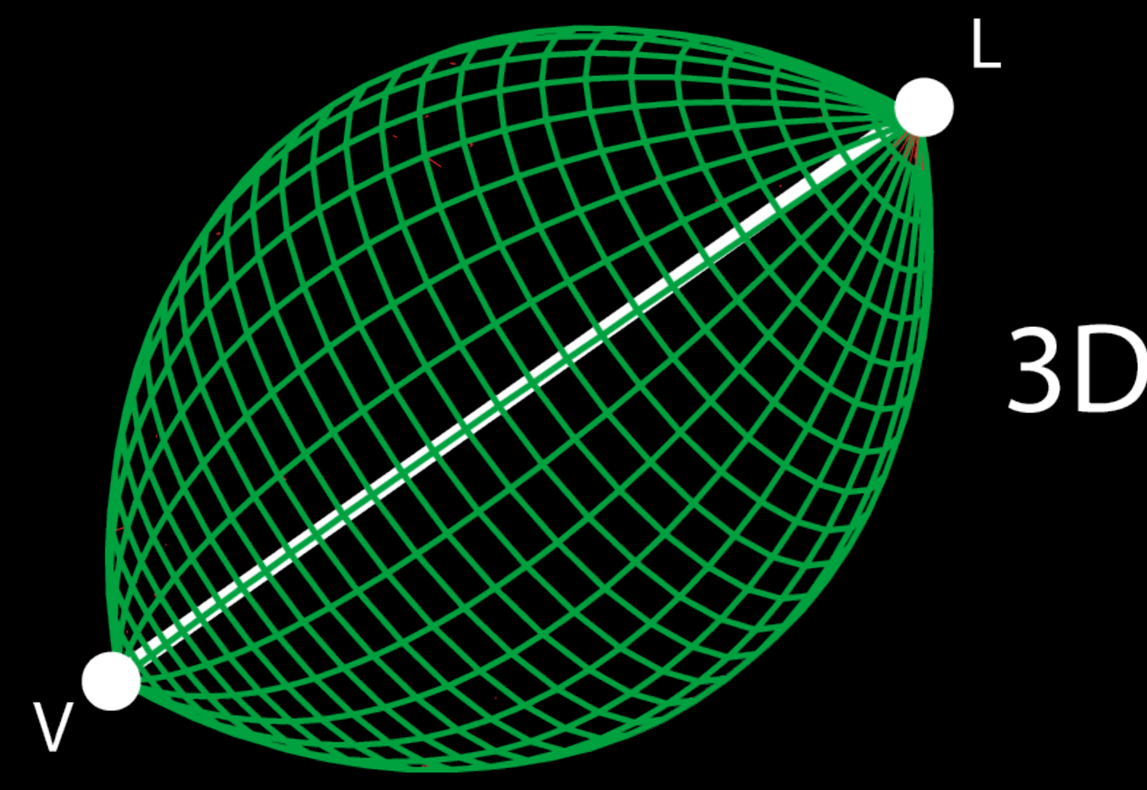
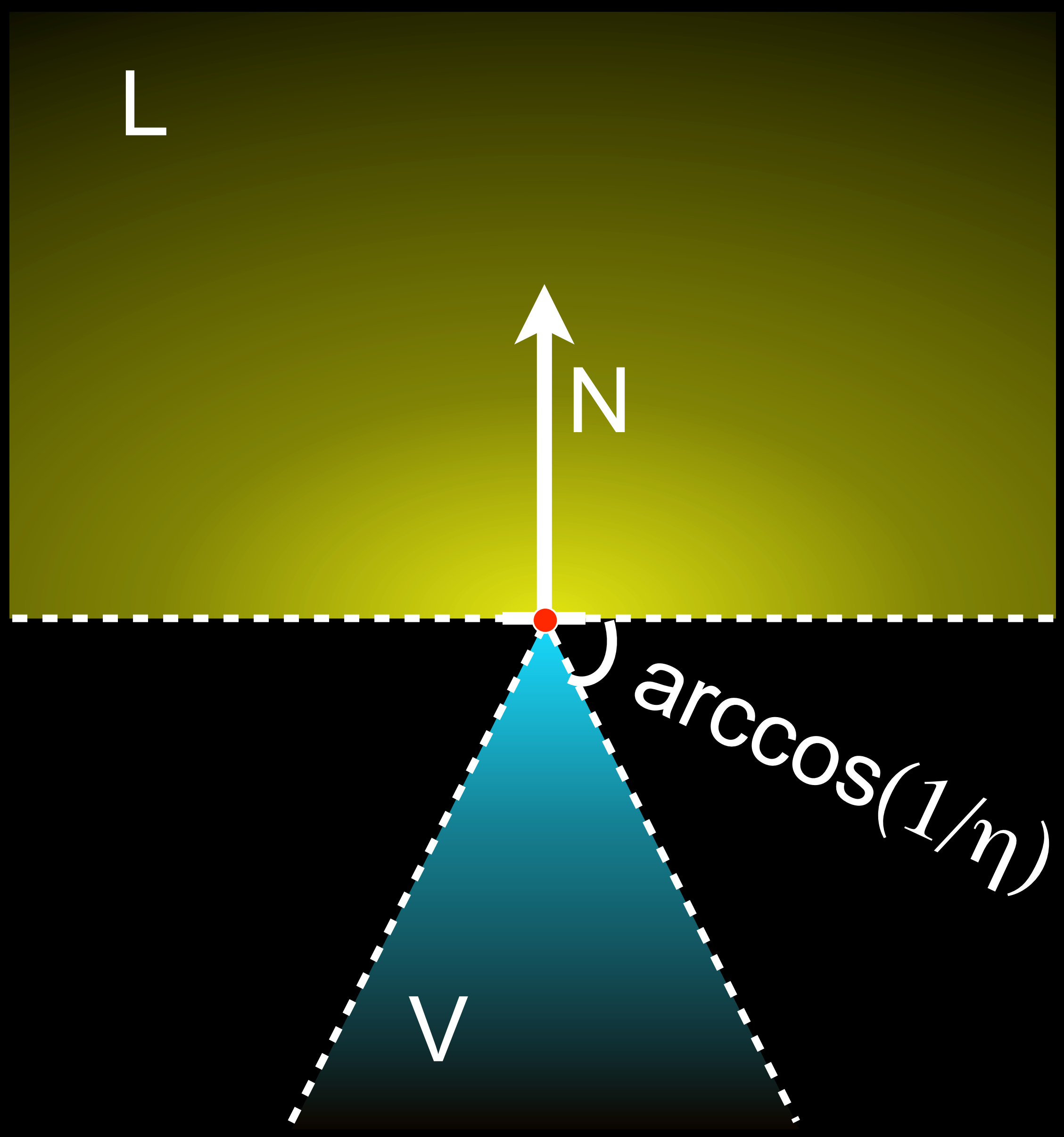




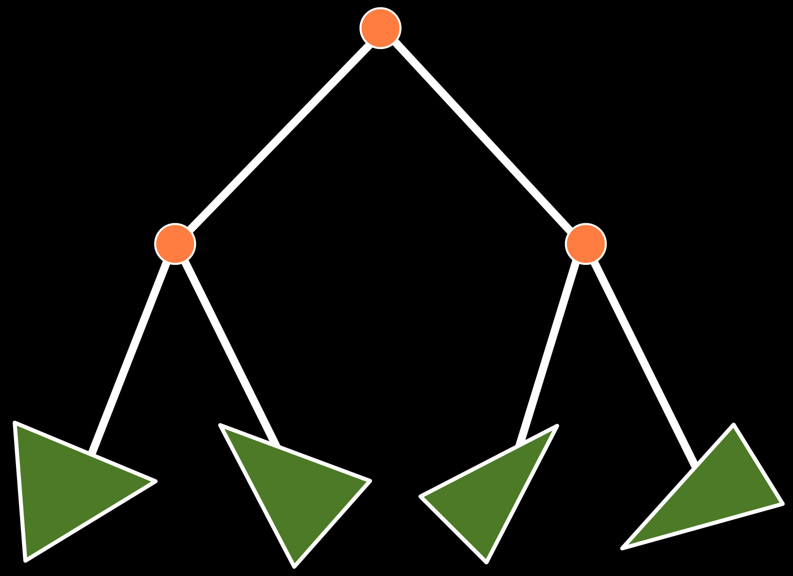
Sidedness Culling Test

- Light L must be on the outside of surface at P
- Receiver V must be inside within the critical angle



Hierarchical Culling for Meshes

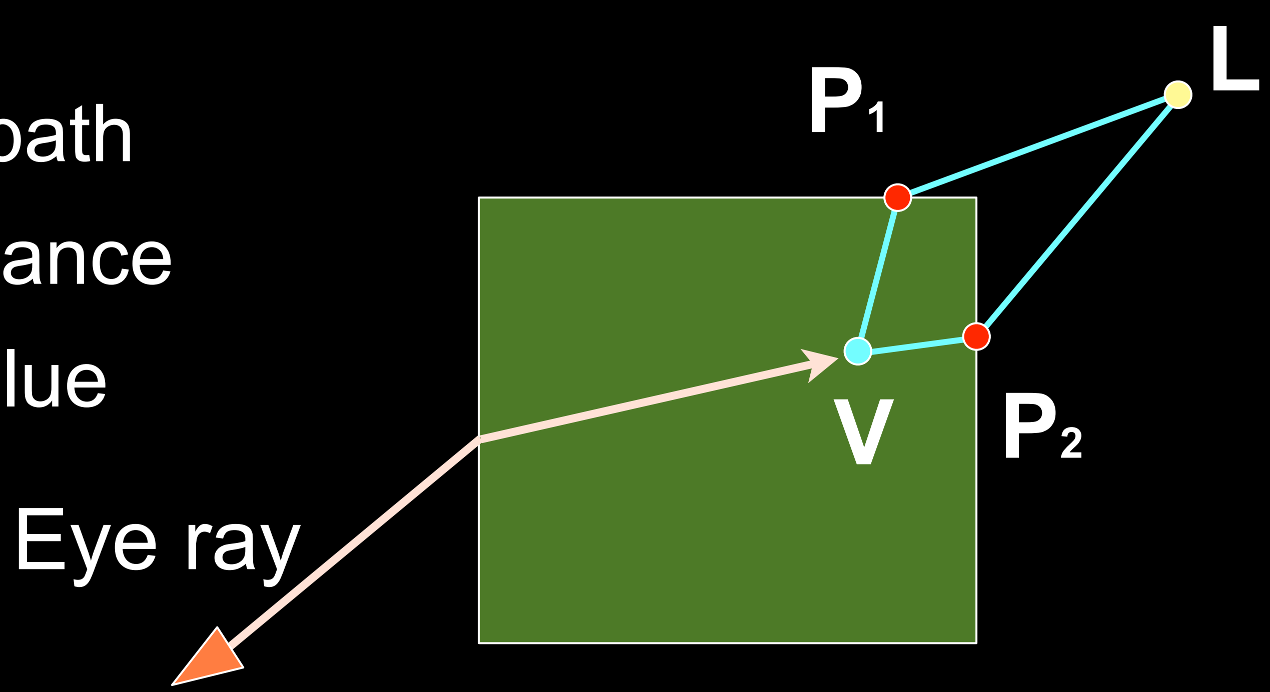
- Apply culling test to groups of triangles
- Use 6D position-normal tree [Bala et al. 03]
  - Leaves are boundary triangles
  - Boxes and cones bound positions and normals
  - Traverse top-down



Position-normal tree

Algorithm Summary

- Build position-normal tree for each boundary mesh
- For each eye ray
  - Trace until hits surface or volume-scatters at V
  - Select a light source point, L
  - Traverse tree to solve for all P on boundary
  - For each solution point P
    - Check for occlusion along path
    - Compute effective light distance
    - Add contribution to pixel value



Effective Distance to Source

- Refraction alters usual $1/r^2$ intensity falloff
  - –Can focus or defocus the light
- Compute effective light distance for each path
  - –Simple formula for triangles without shading normal
  - –Use ray differentials [Igehy 99] for shading normal case
  - –See paper for details

Outline

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- Hierarchical culling for meshes
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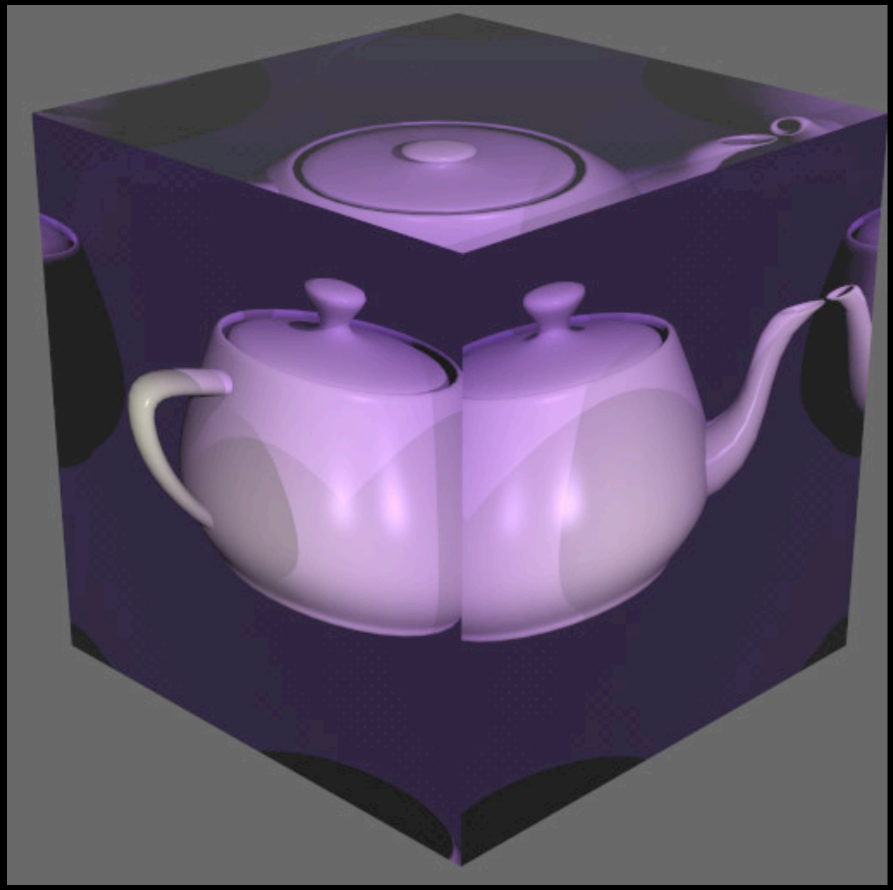
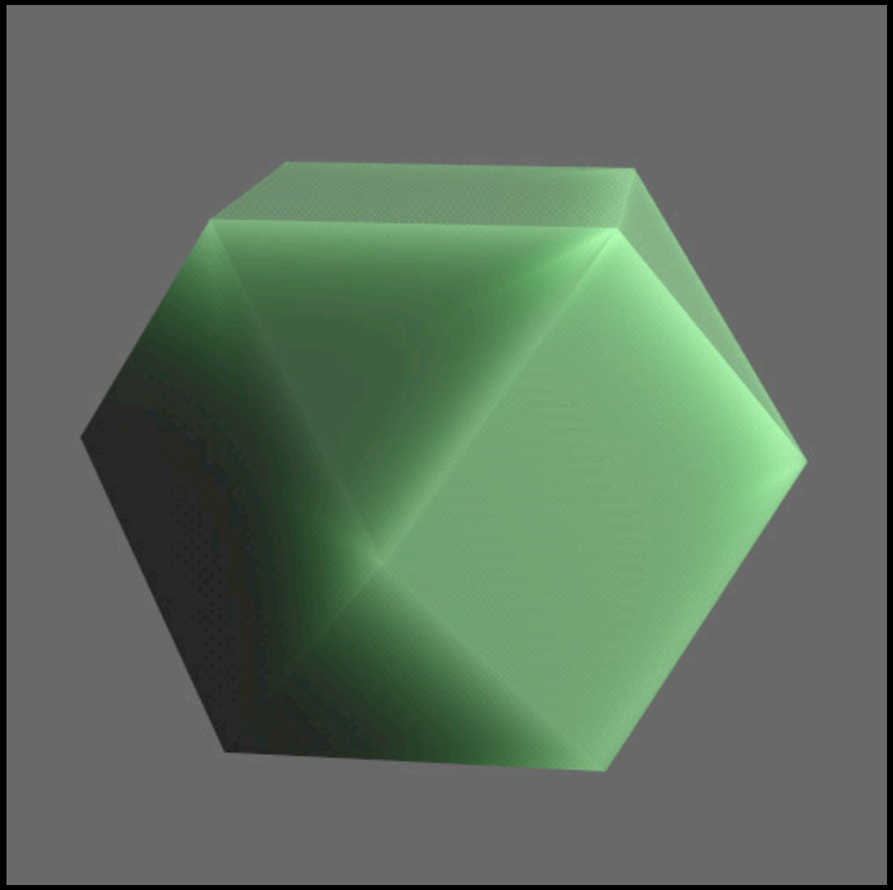
Results - CPU

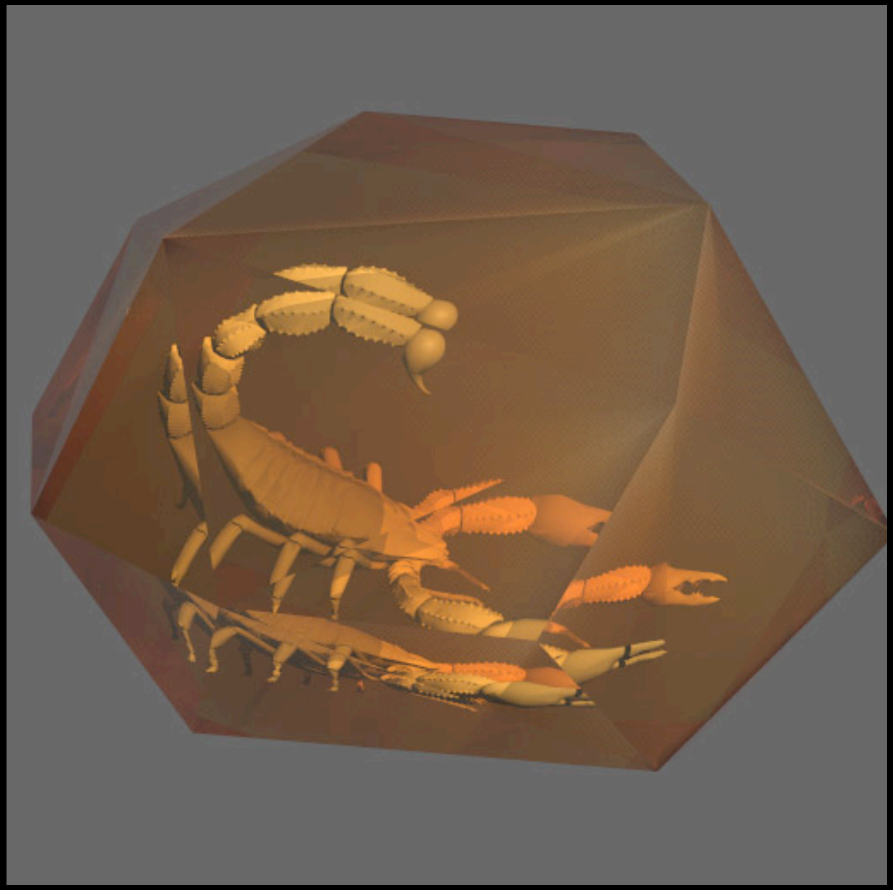
- Three scenes without shading normals



Teapot



Cuboctahedron



Amber

Results - CPU

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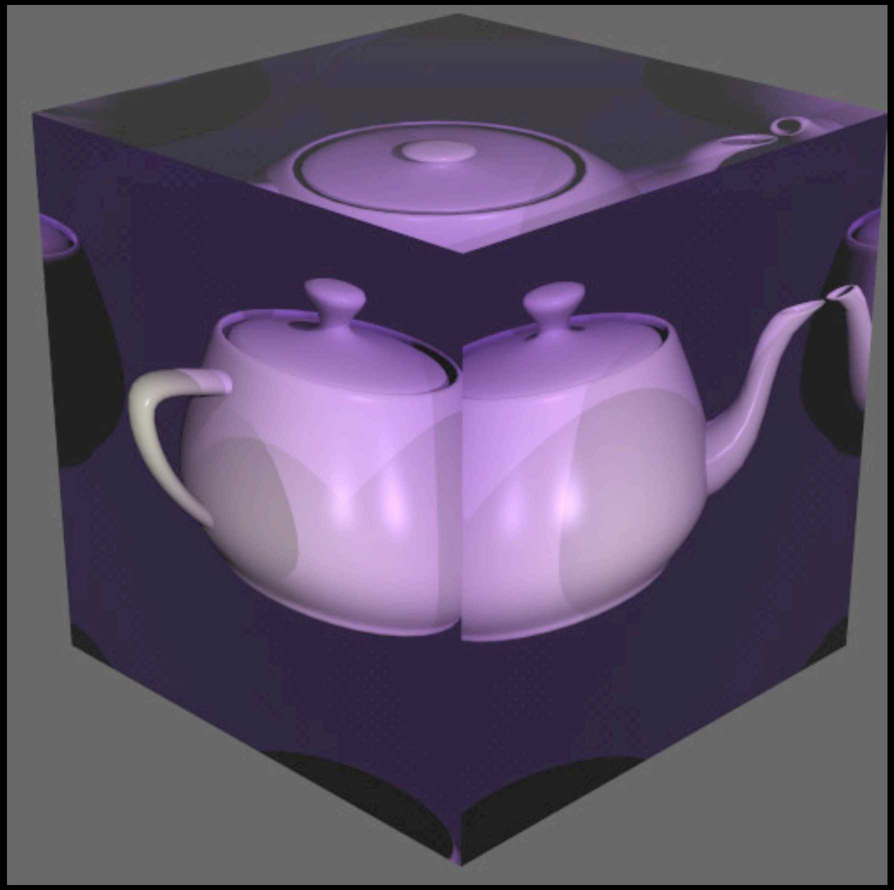
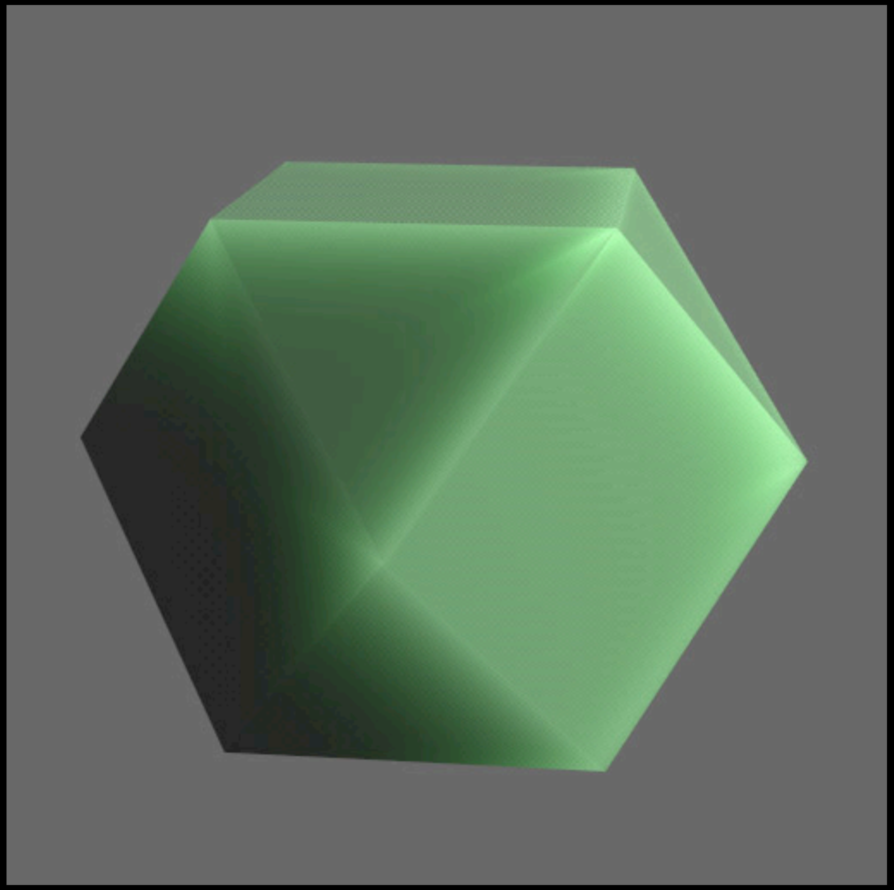
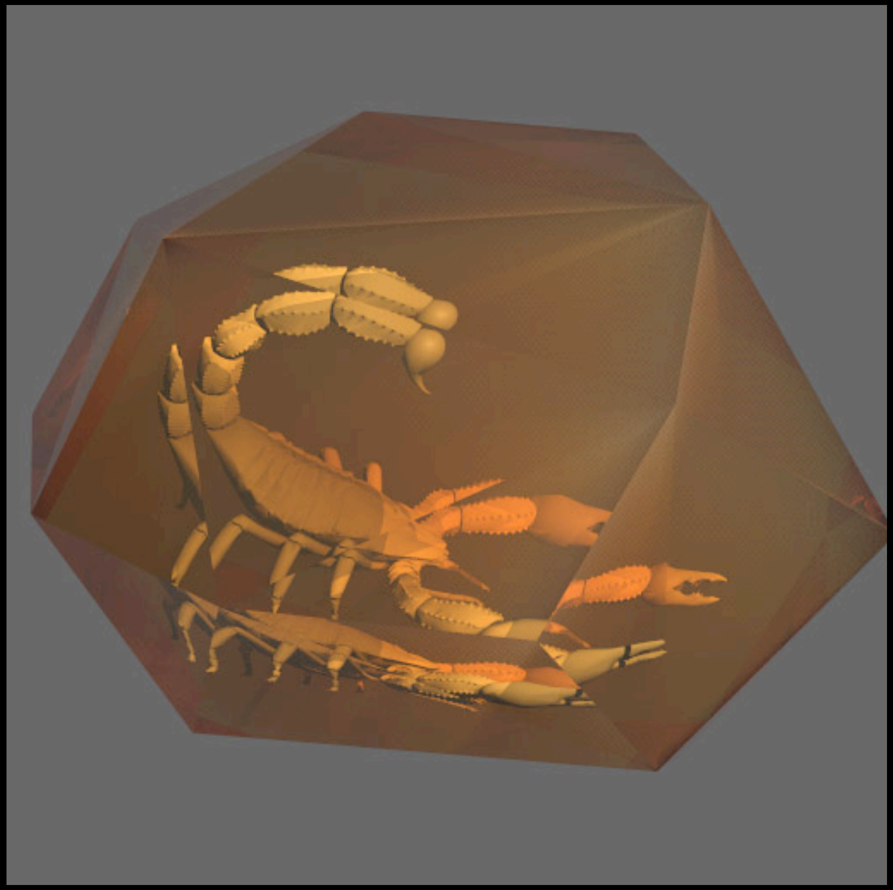


Teapot
15.3s



Cuboctahedron
13.9s

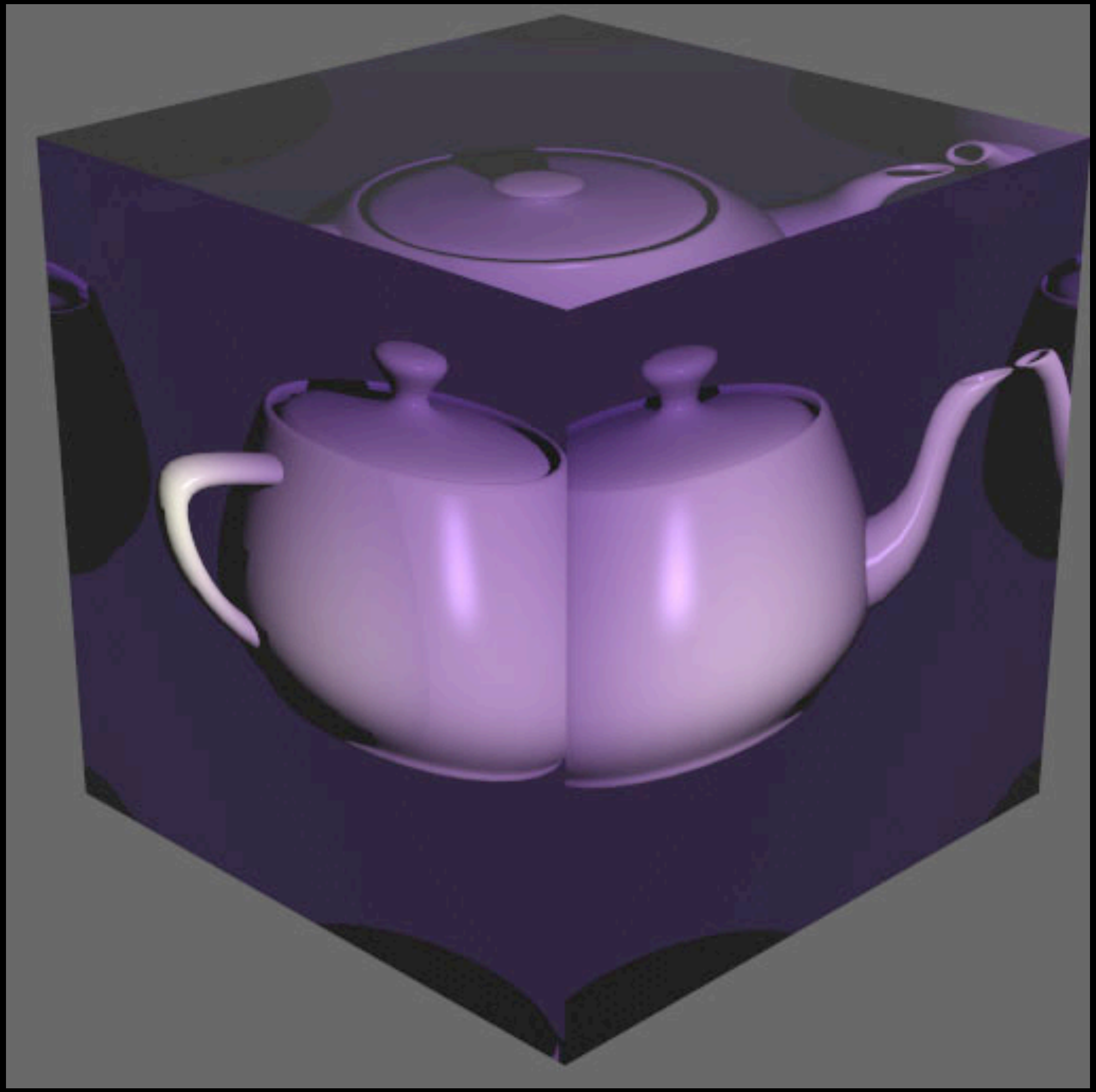


Amber
19.2s

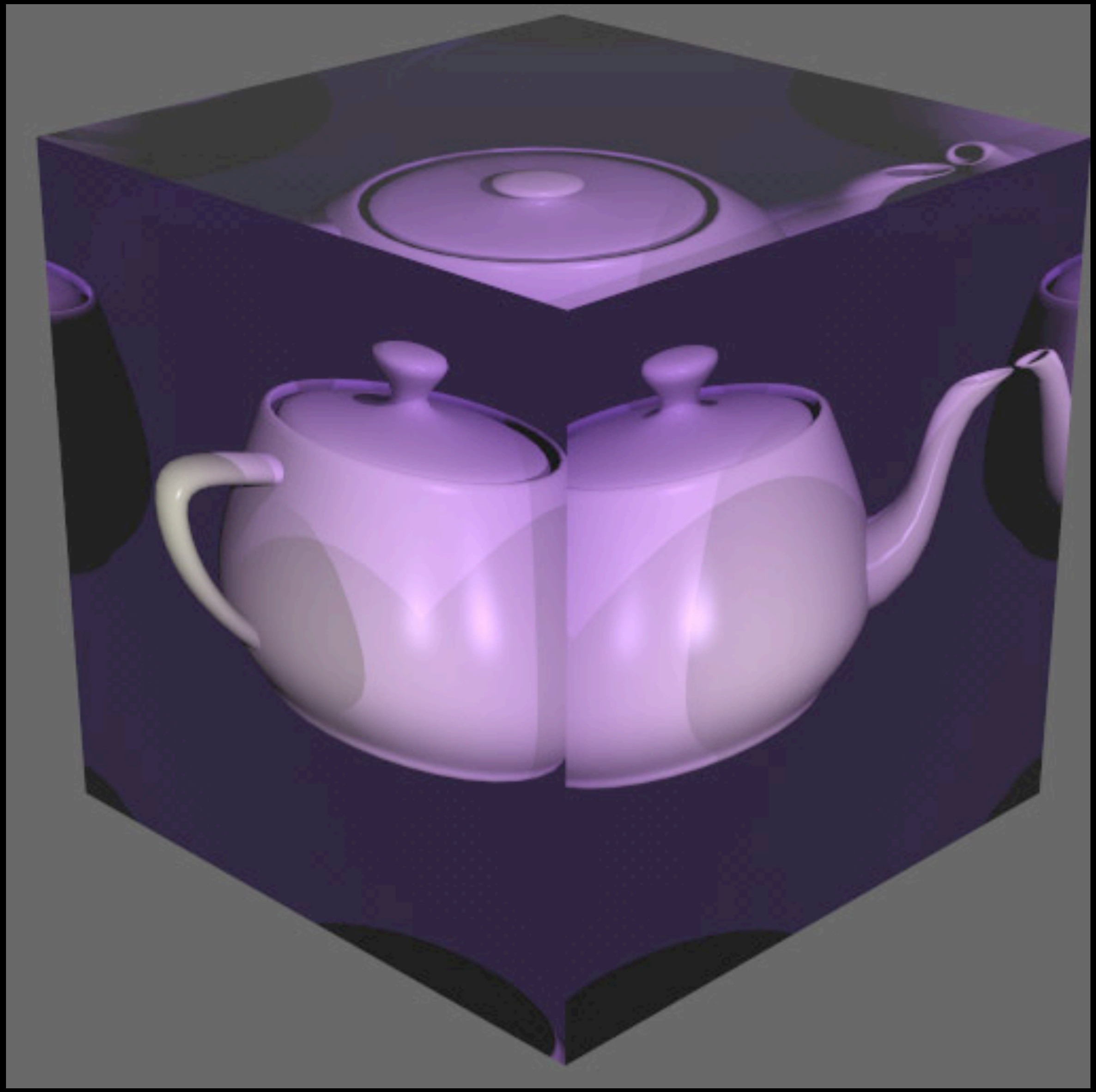
512x512 images, 64 samples per pixel, 8-core 2.83GHz Intel Core2

Results - Teapot

- Teapot quality comparison



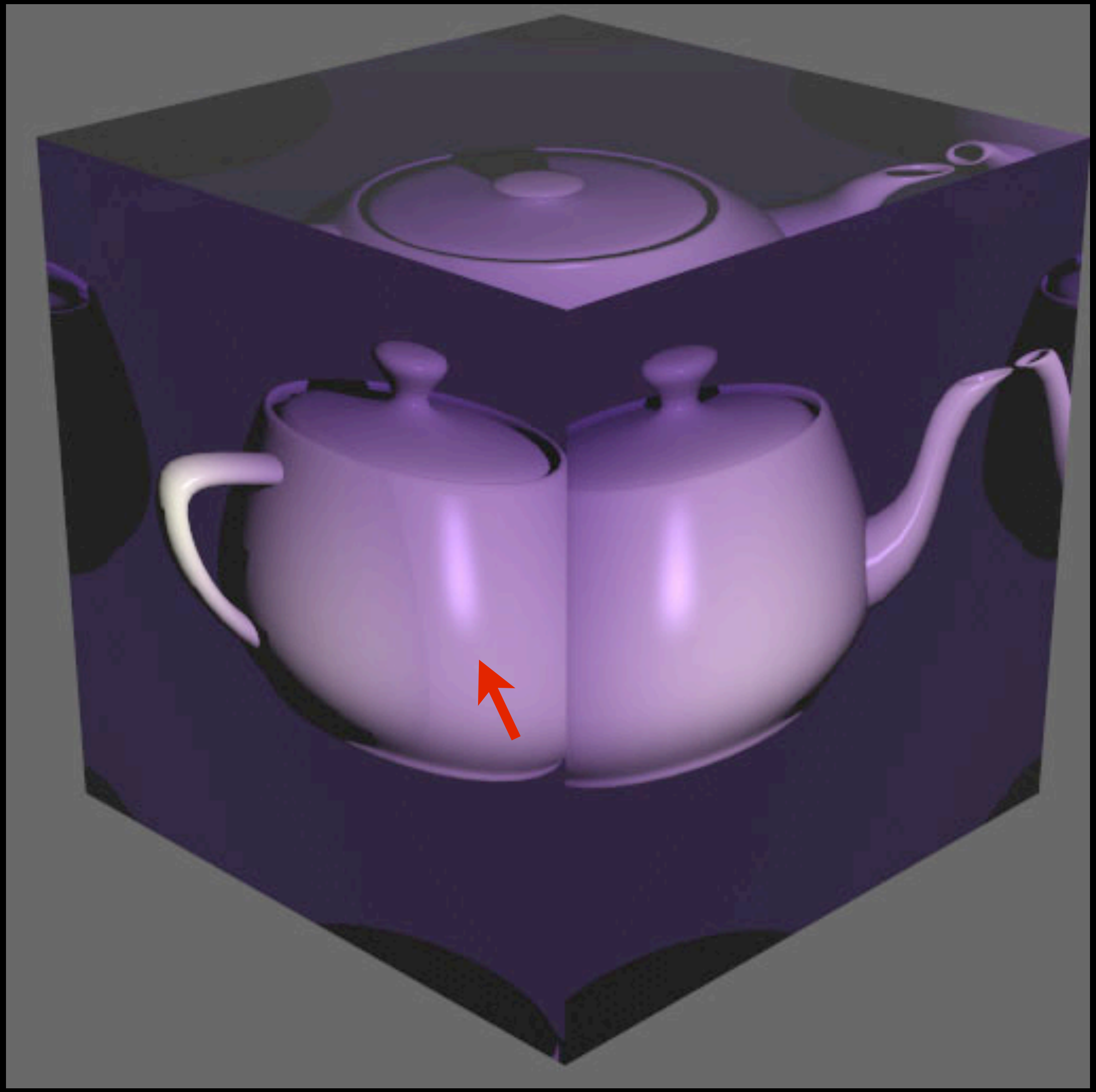
Shadow rays ignore refraction



Our method (15.4s)

Results - Teapot

- Teapot quality comparison



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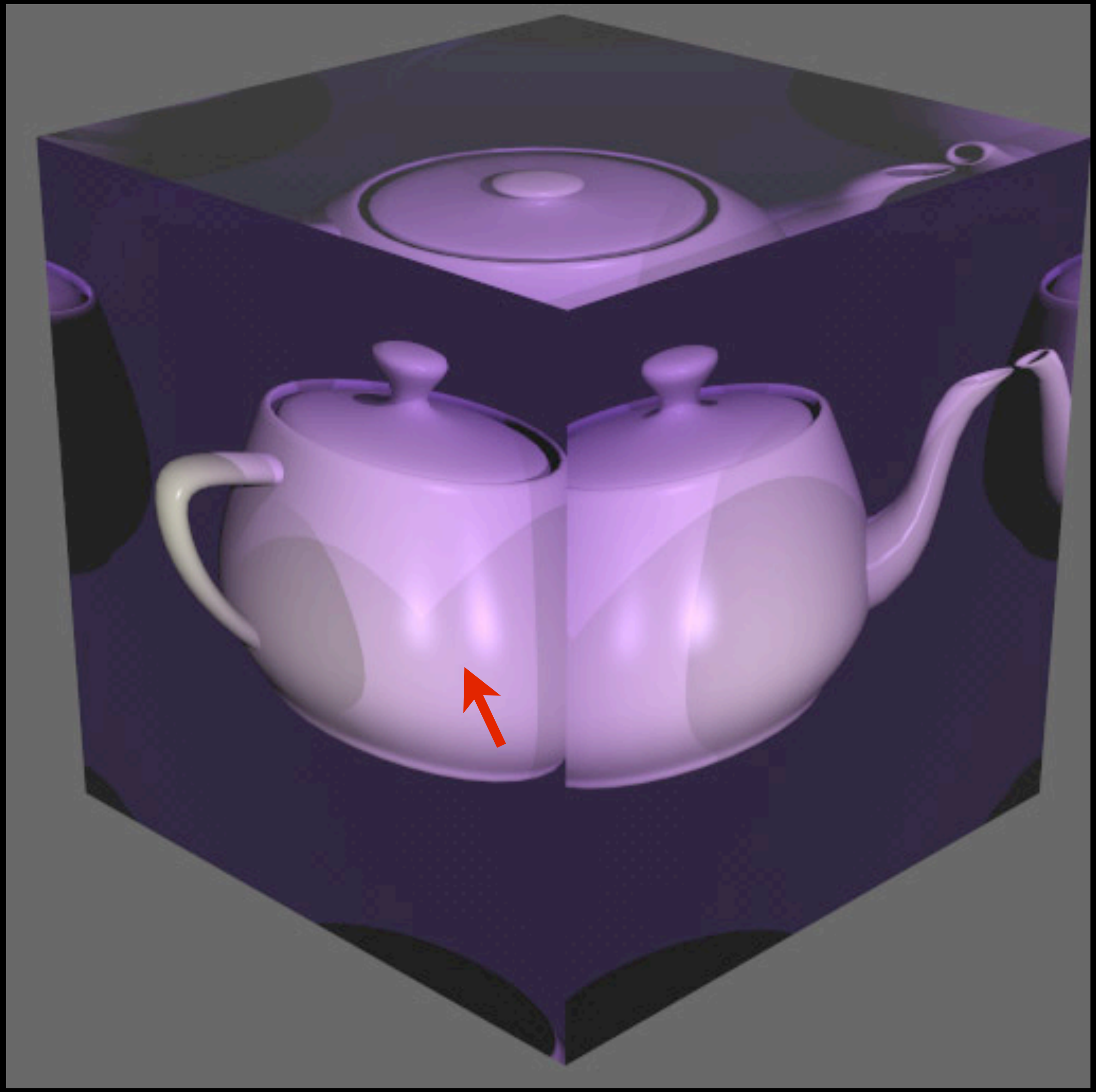
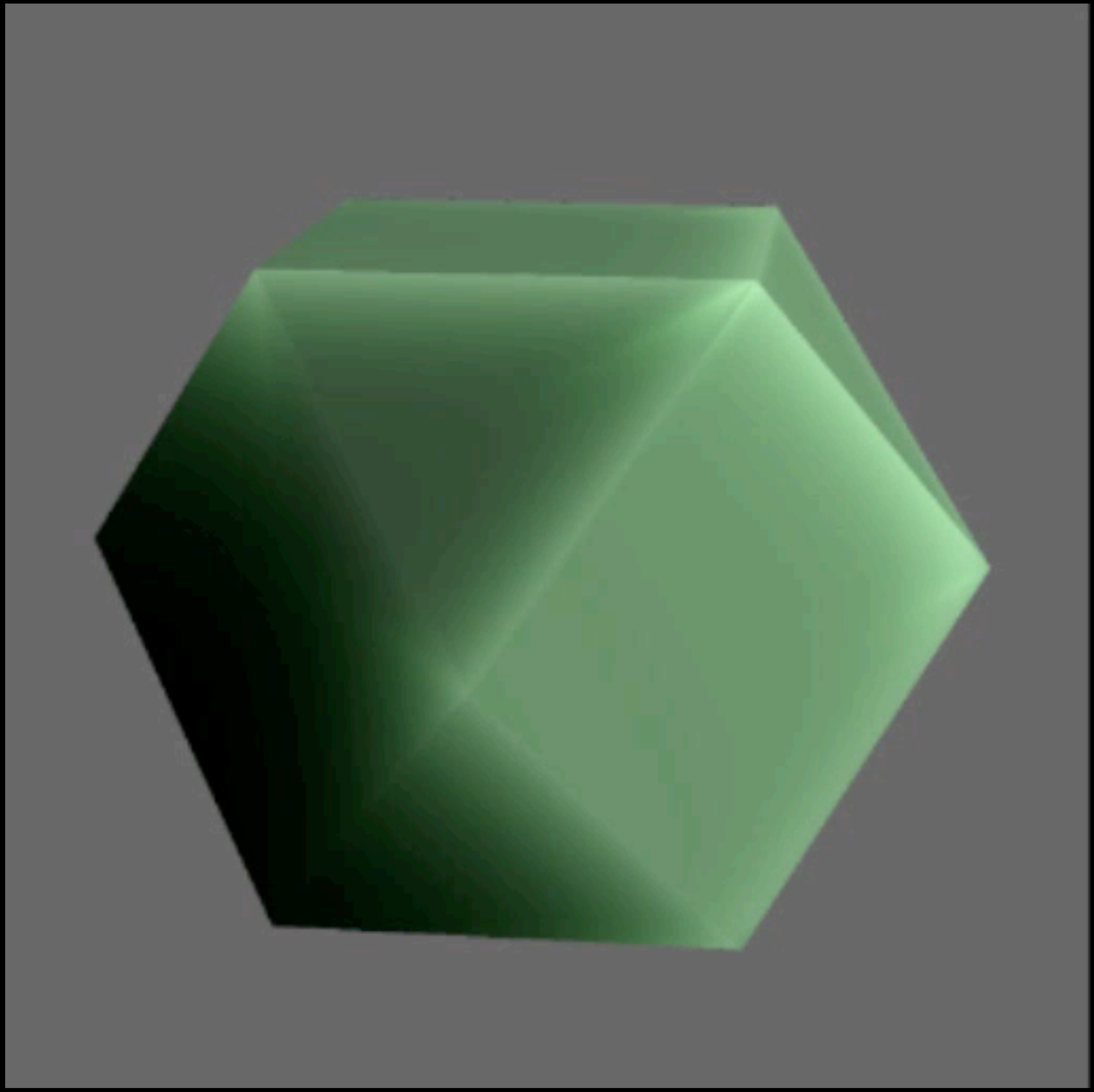


Our method (15.4s)

Results - Cuboctahedron

- Cuboctahedron movie (13.9s)



Results - GPU

- Implemented on GPU using CUDA 2.0
  - 1D, 2D Newton iteration
  - Hierarchical pruning
  - Ray tracing based on [Popov et al. 2007]
  - One kernel thread per eye ray
  - Does not yet support all scenes

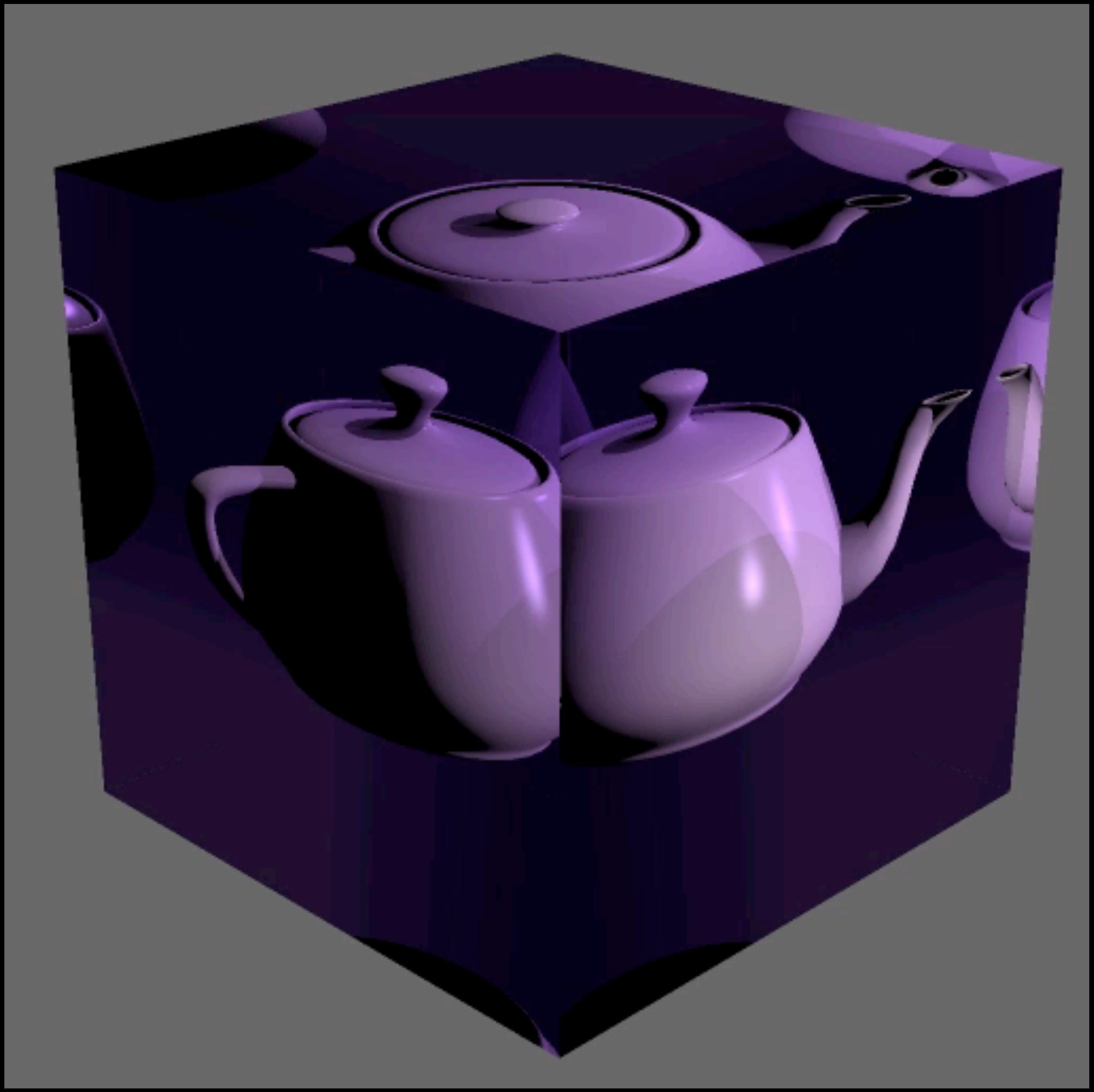
Results - GPU

Name	Render Time	FPS
Teapot	0.1 s	10 fps
Cuboctahedron	0.14 s	7 fps
Amber	0.3 s	3 fps

512x512 images, 2 eye rays per pixel + 40-60 volume samples, nVIDIA GTX 280, CUDA 2.0

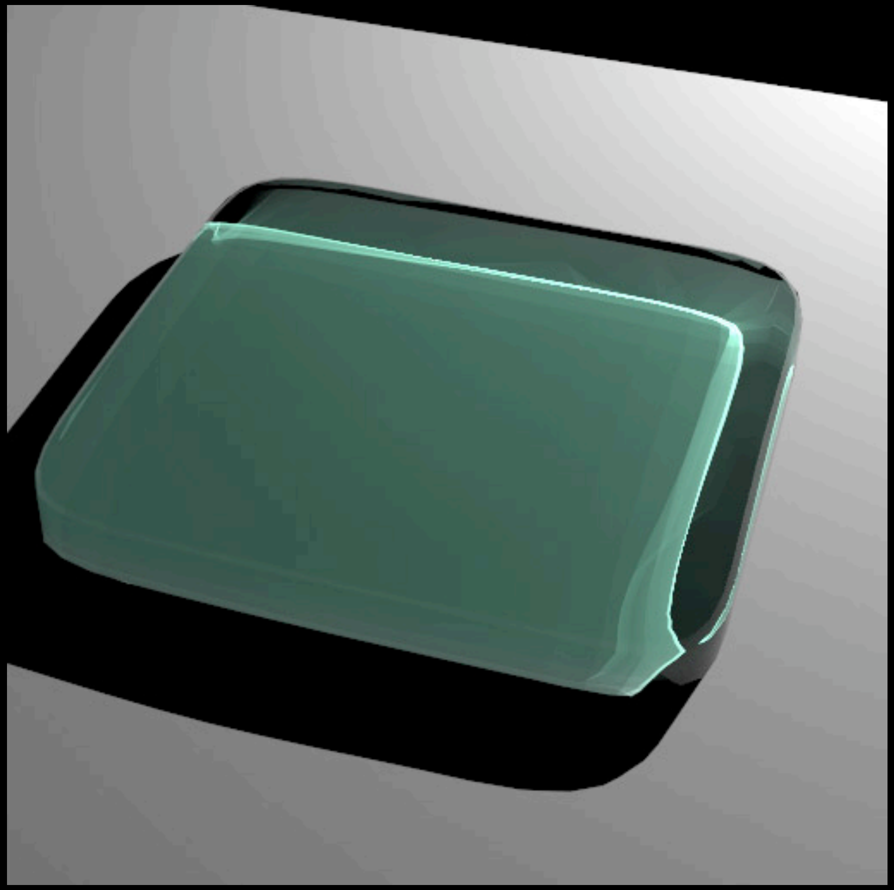
Results - GPU

- Teapot example
  - 10 fps on GPU

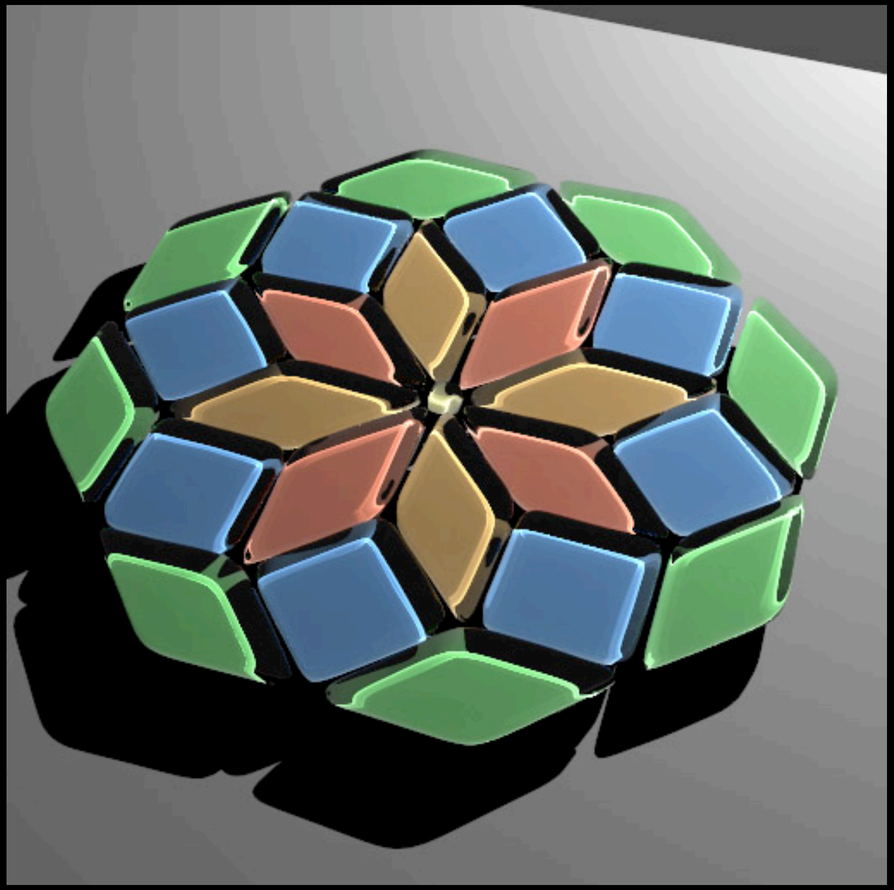


Results - CPU

- Three scenes with shading normals



Glass tile

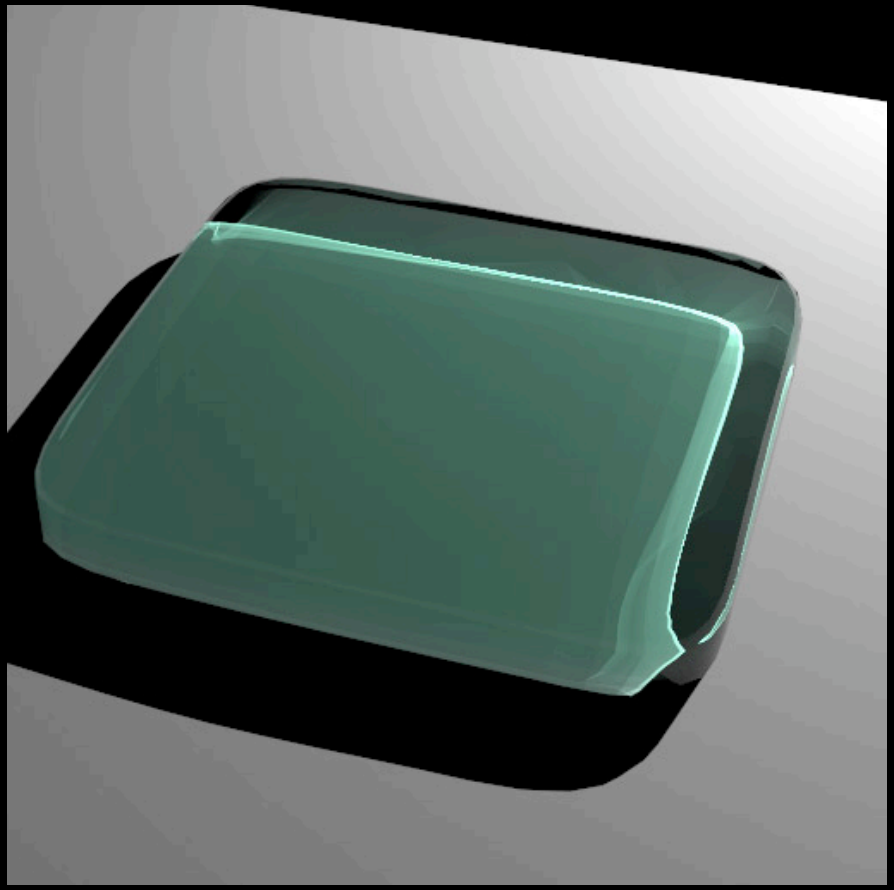


Glass mosaic

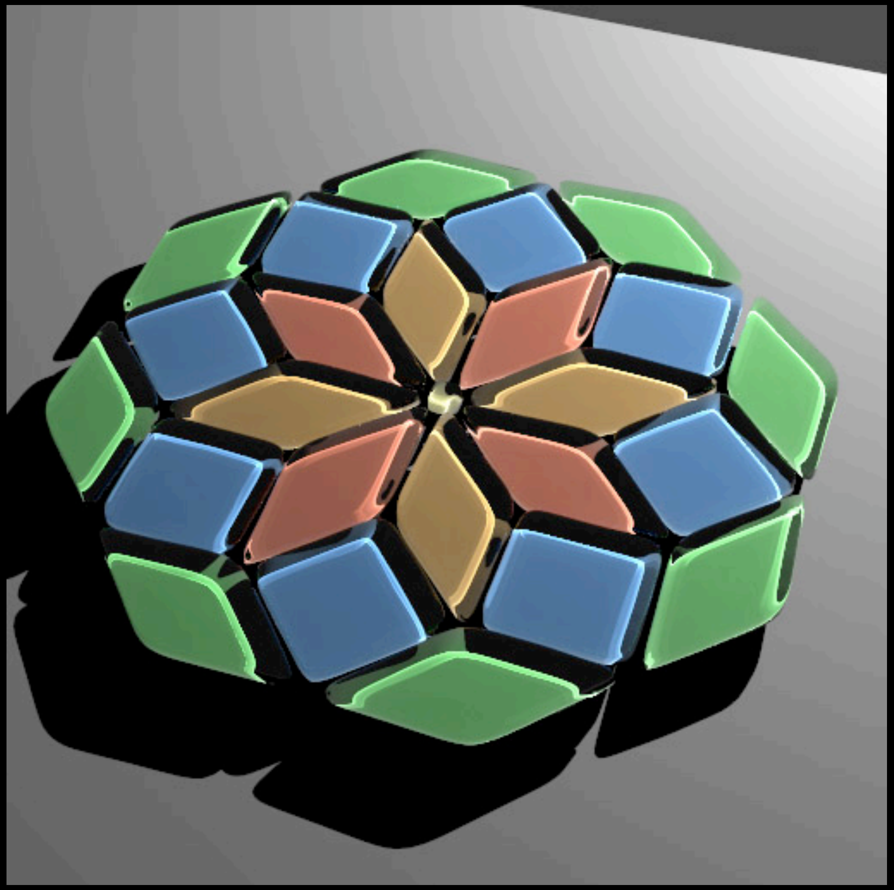
Pool

Results - CPU

- Three scenes with shading normals



Glass tile
66.9s

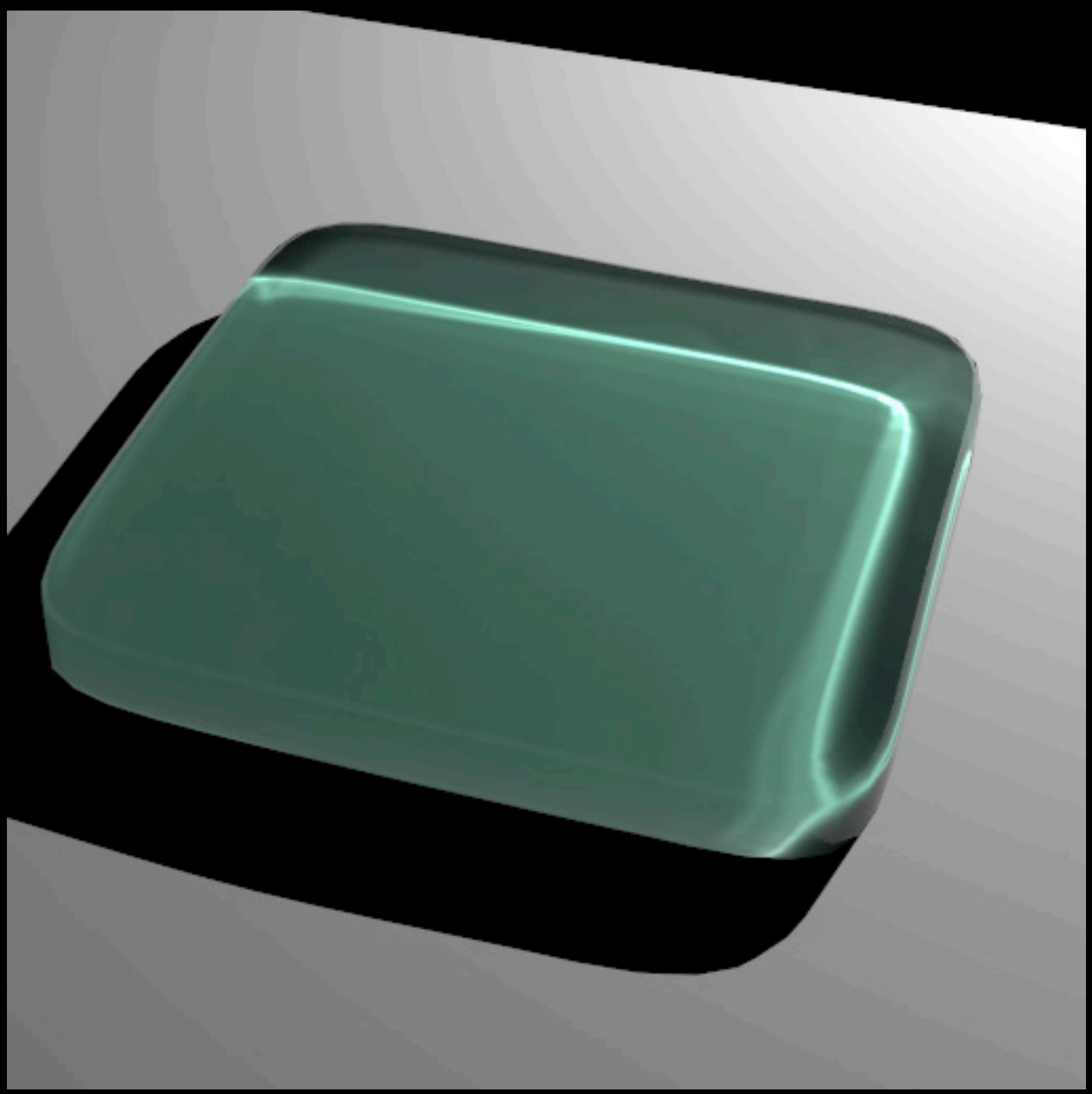


Glass mosaic
87.8s

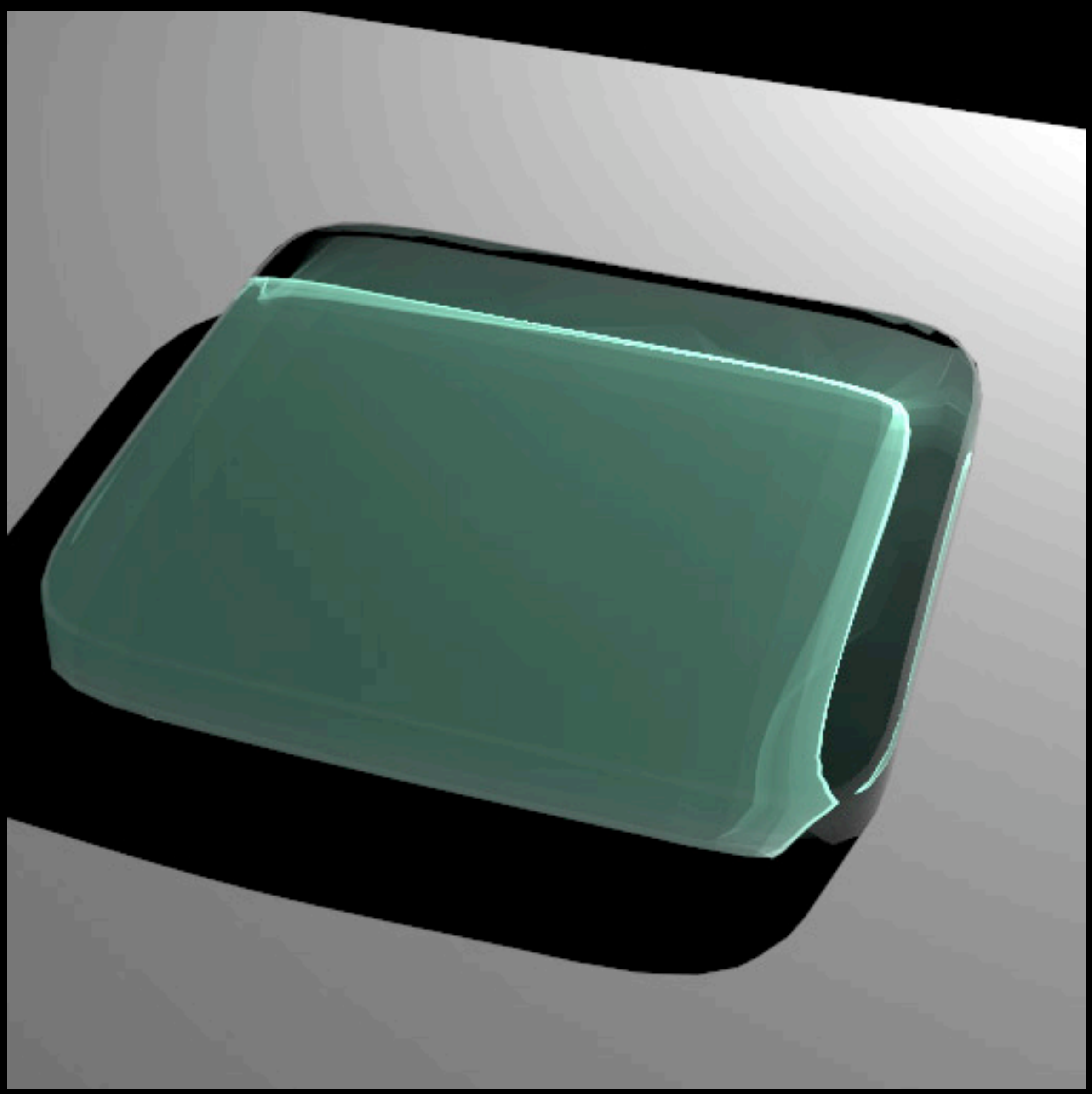
Pool
59.4s

512x512 images, 64 samples per pixel, 8-core 2.83GHz Intel Core2

Results - Glass Tile



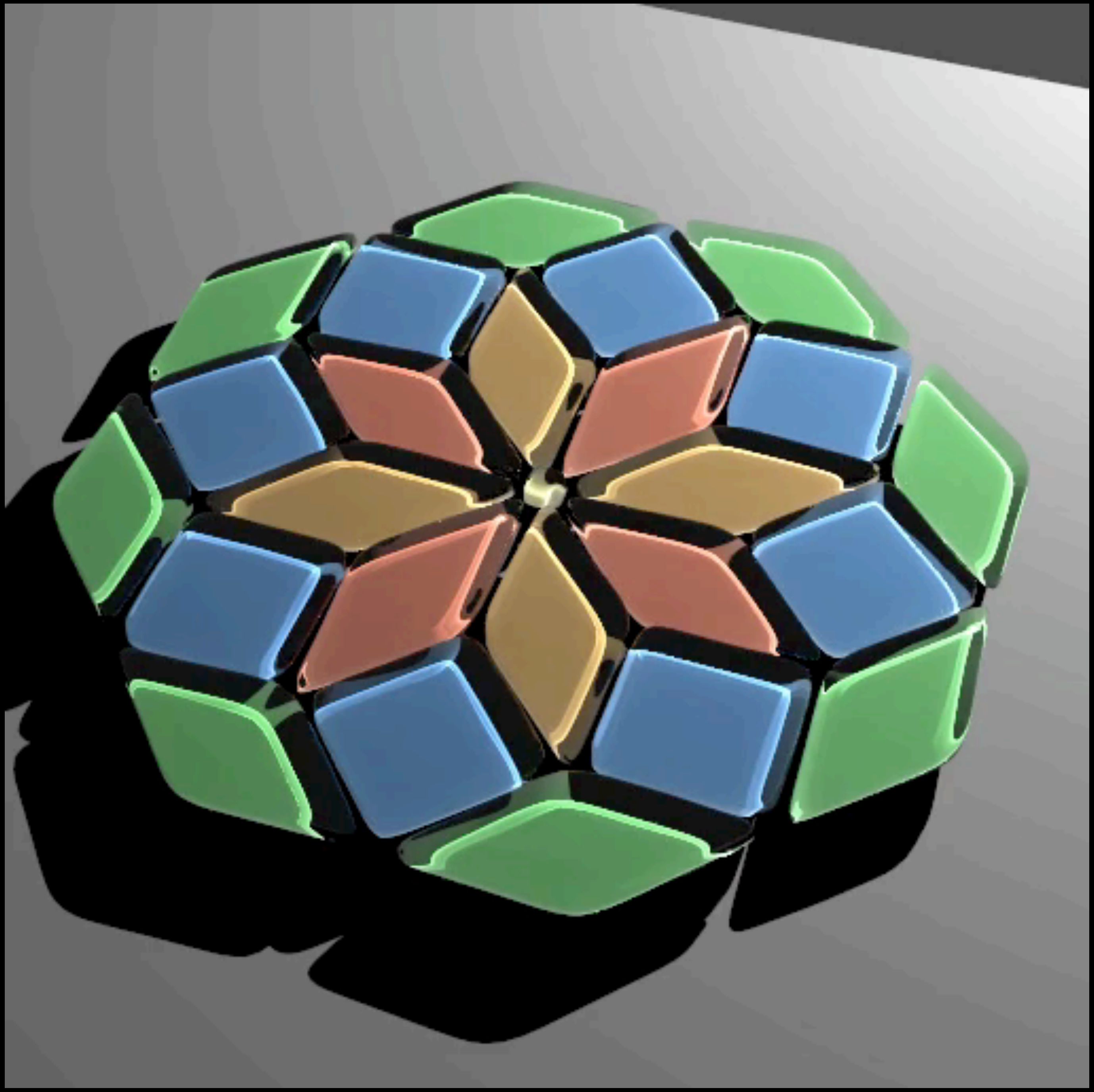
Photon map (equal time)



Our method (66.9s)

Results - Glass Mosaic

- Glass mosaic movie (87.8s)



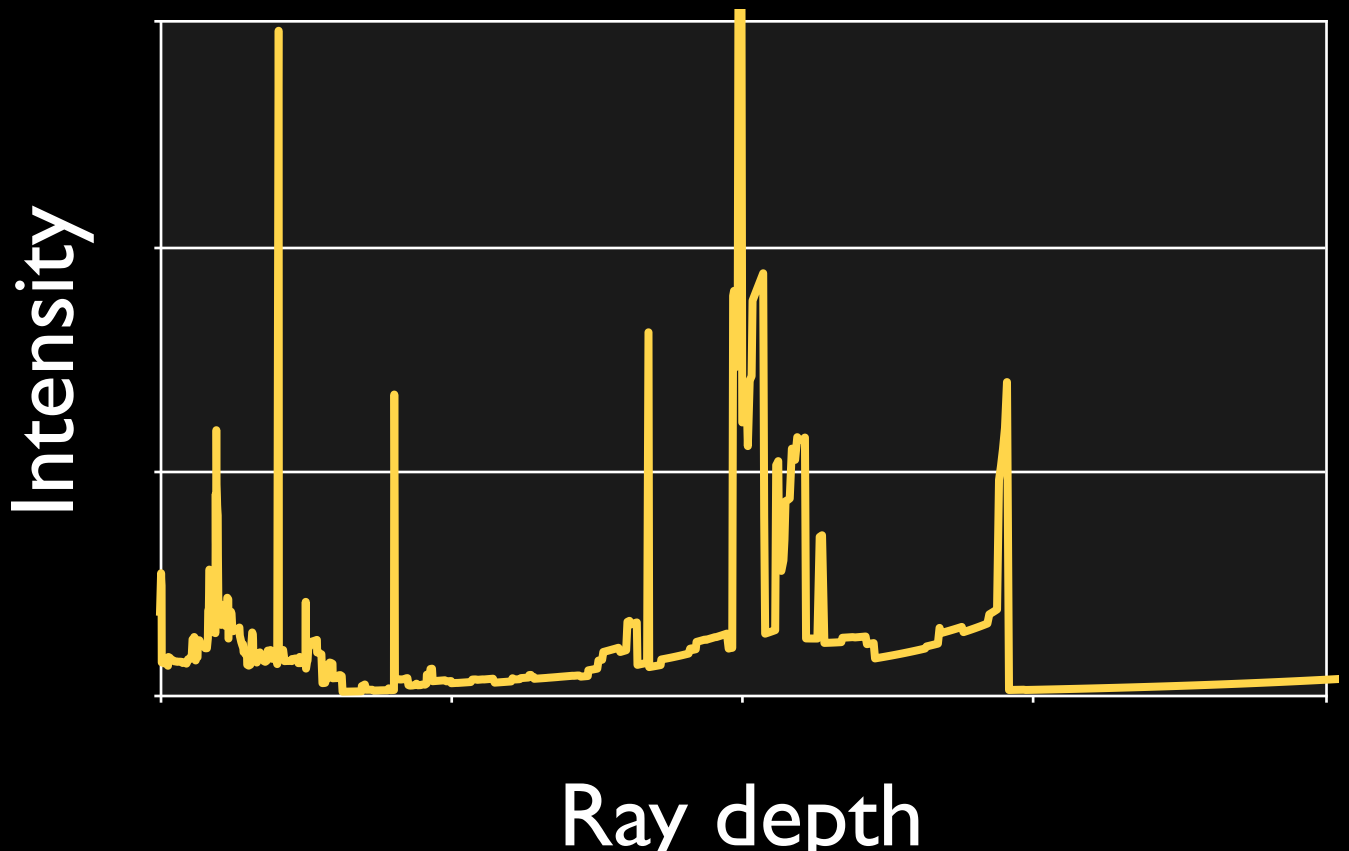
Results - Component Evaluation

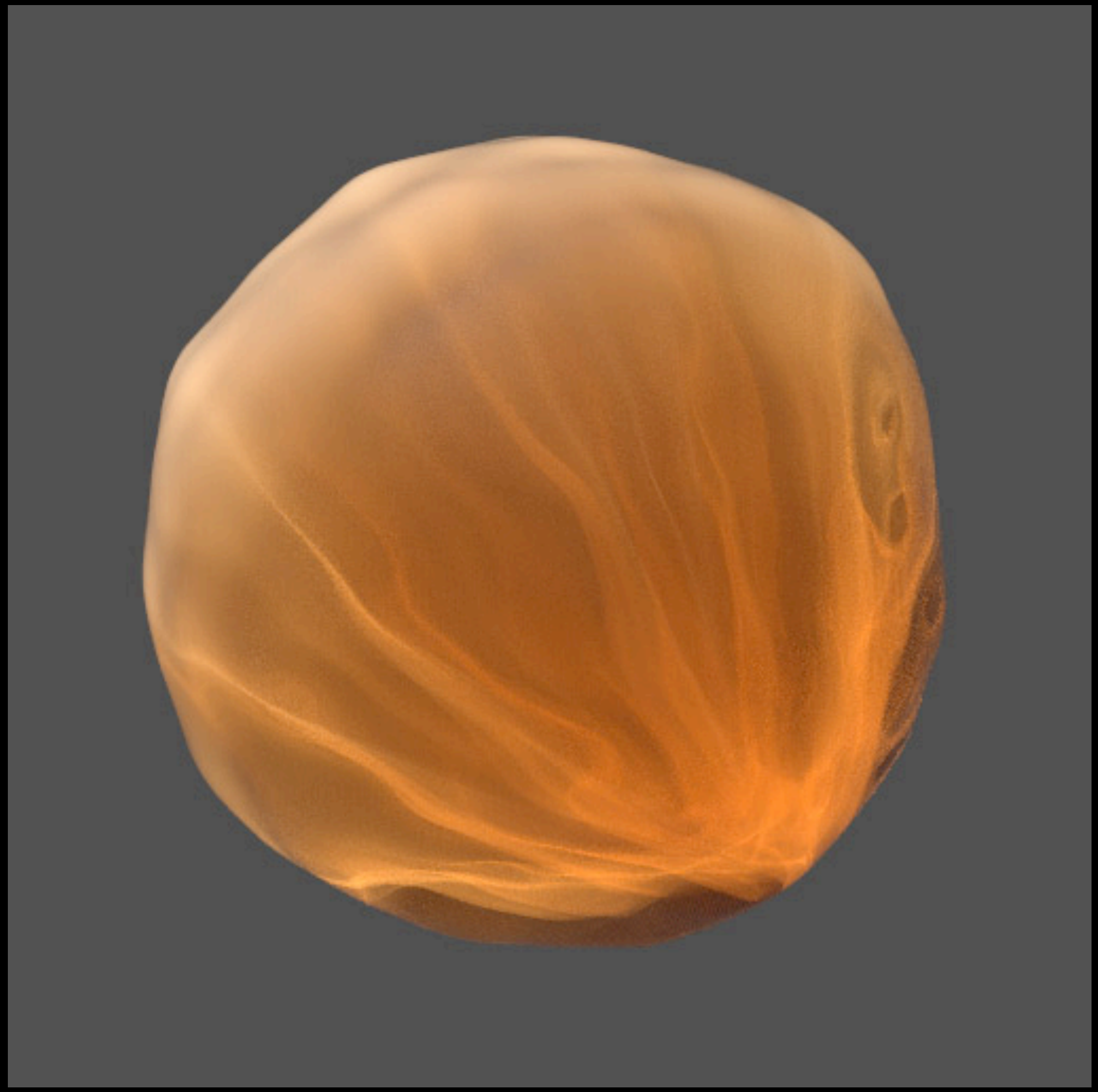
- Evaluation of algorithm components
  - Pool (2632 triangles in boundary)

	Time	Ratio
Without Hierarchy	1934.6s	32x
Guaranteed Convergence	141.1s	2.4x
Subdivision Heuristic	59.4s	1x

Results - Bumpy sphere

- Bumpy sphere (9680 triangles)
  - Volume sampling noise
    - Used 128 samples per pixel
    - Effective distance clamping



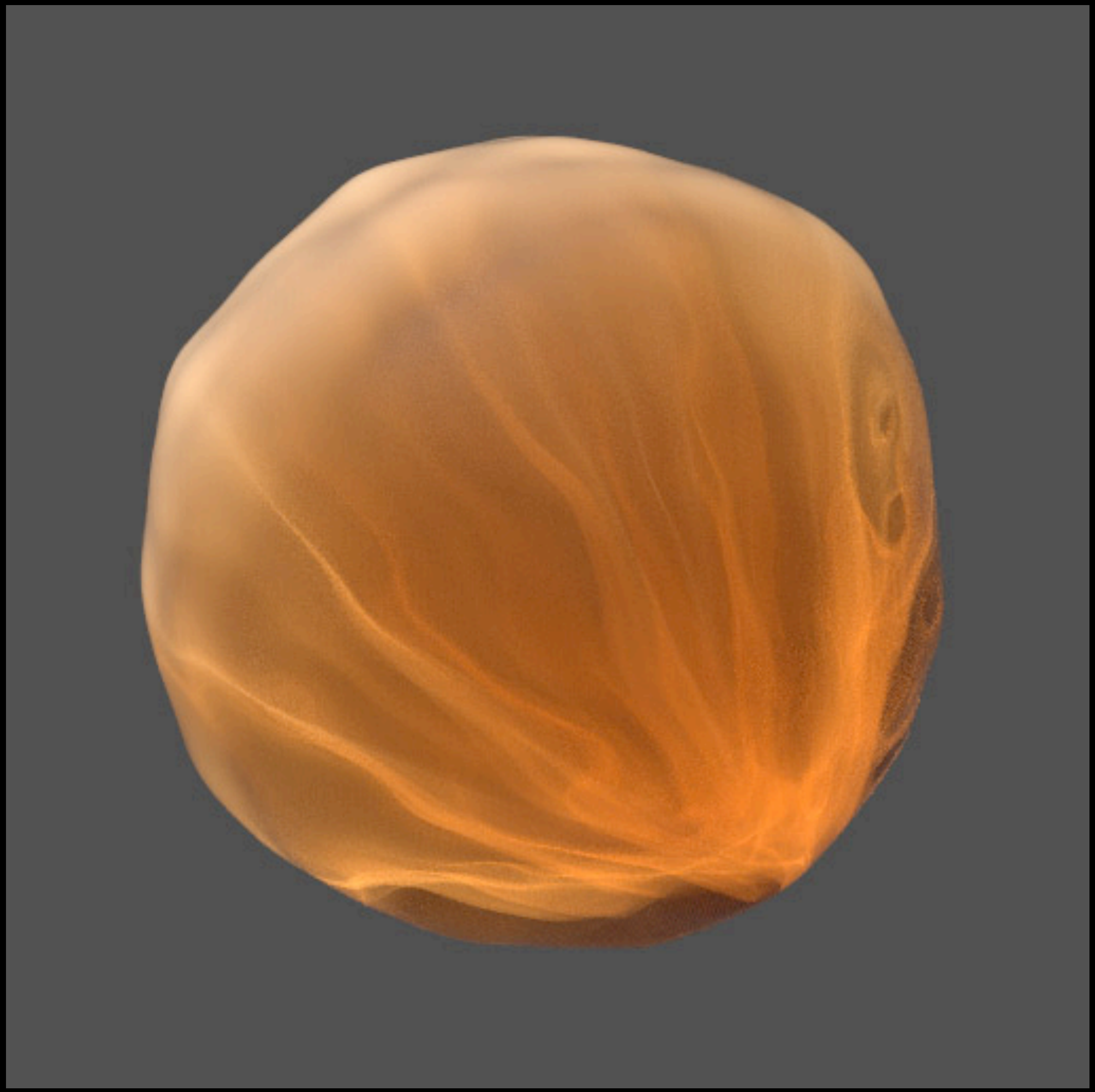


Our method 304.3 s

Results - Bumpy sphere

- Bumpy sphere (9680 triangles)

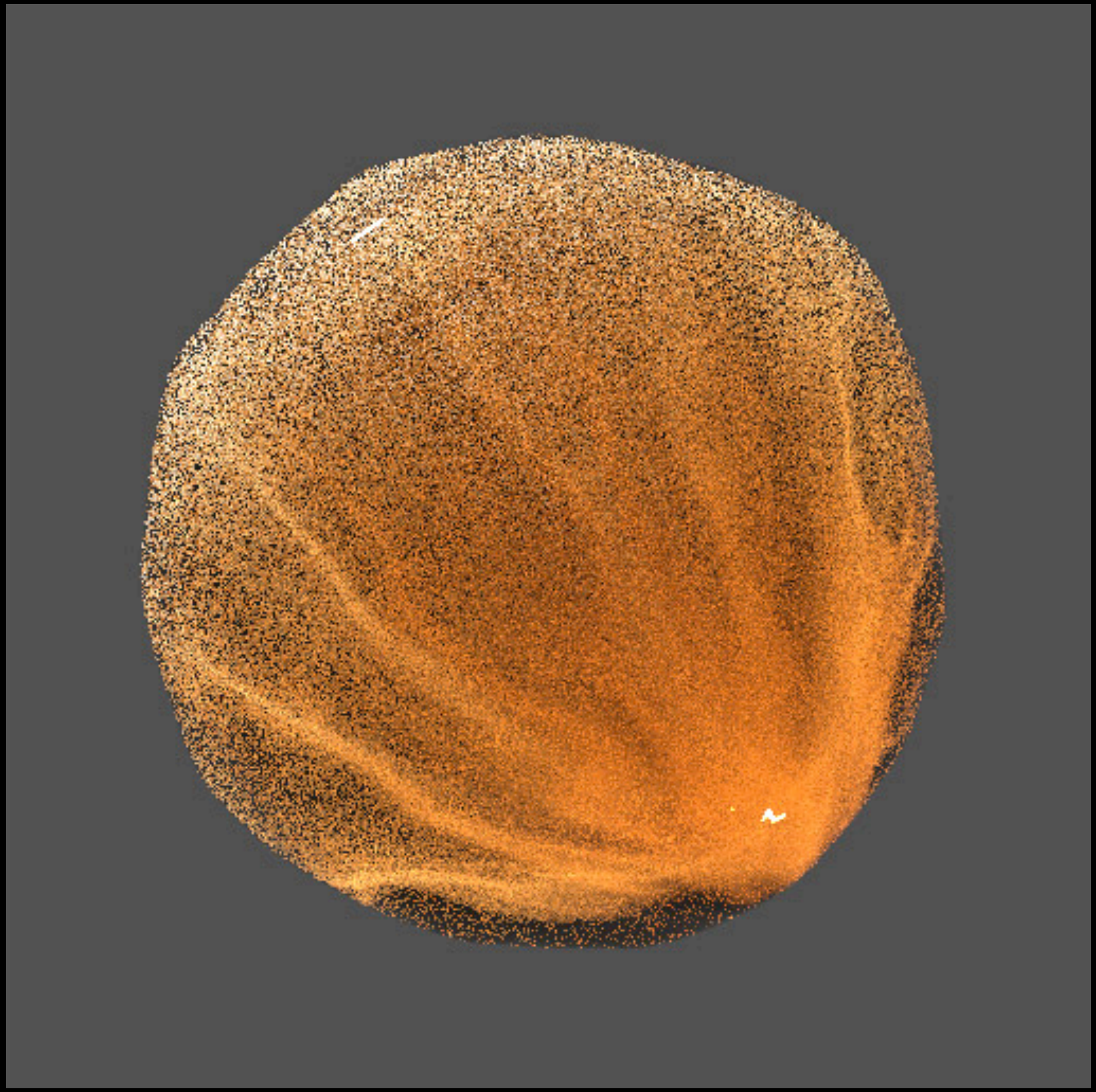
Shadow rays ignore refraction



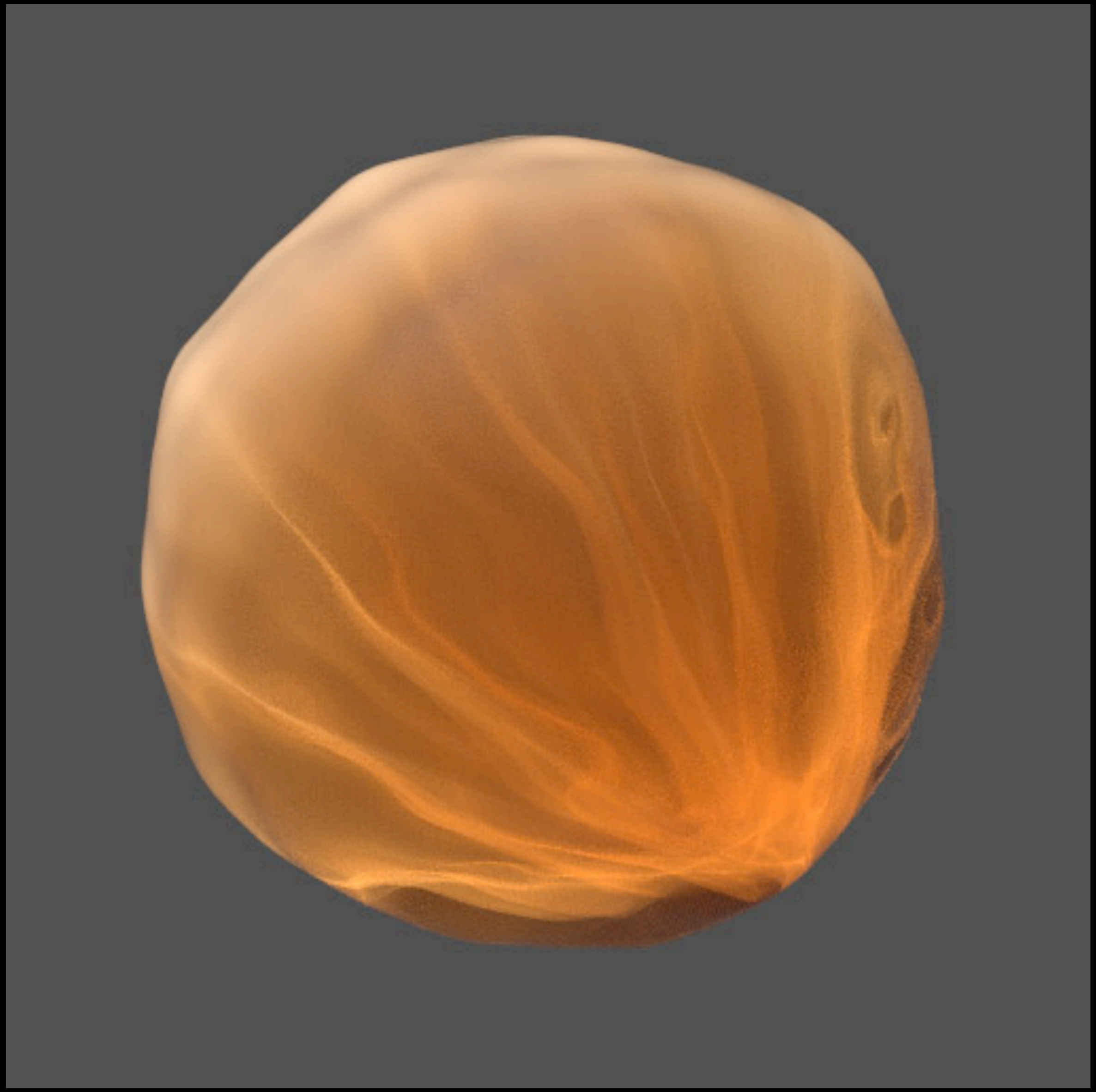
Our method

Results - Bumpy sphere

- Bumpy sphere (9680 triangles)



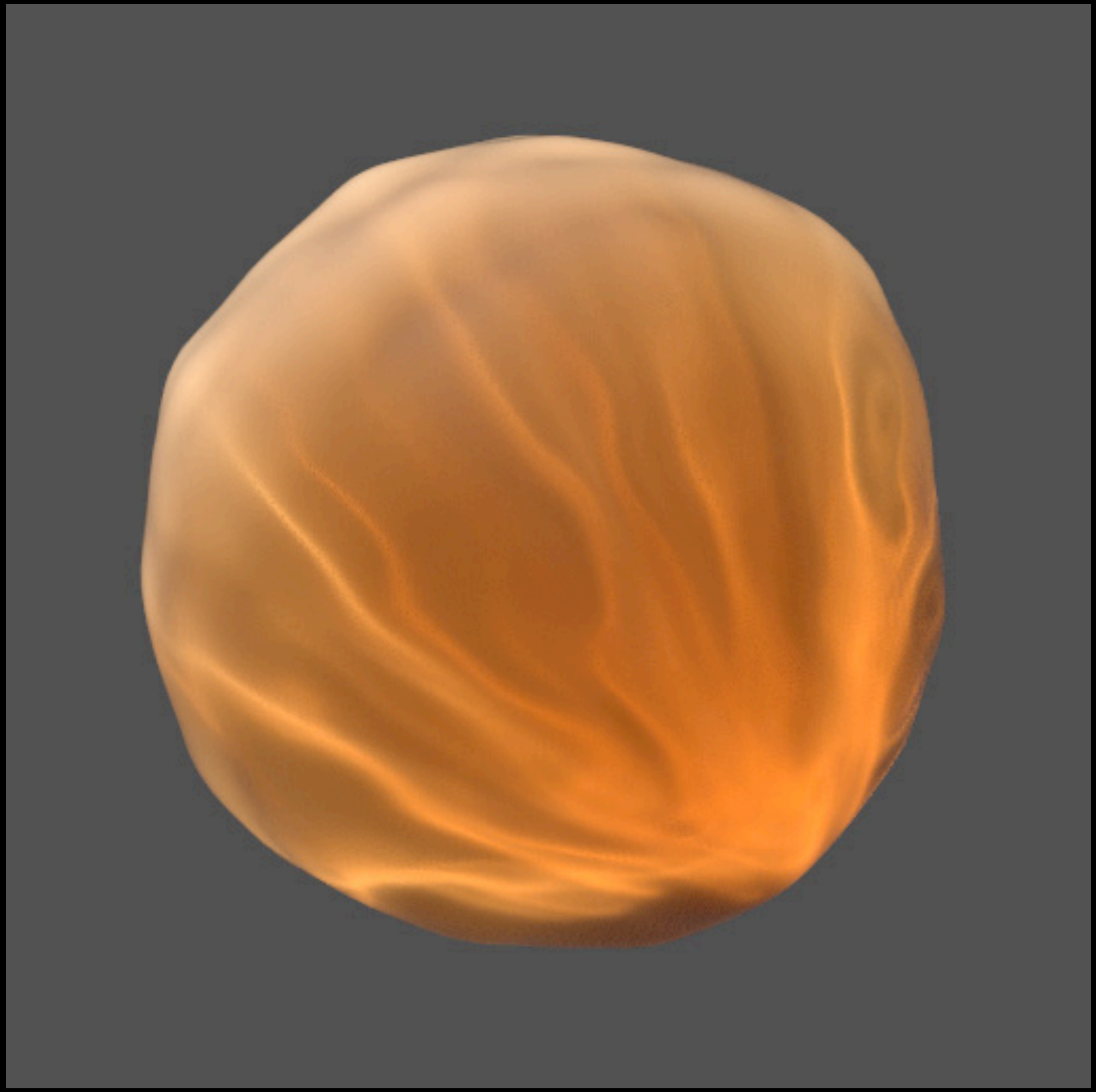
Path tracing(16x time)



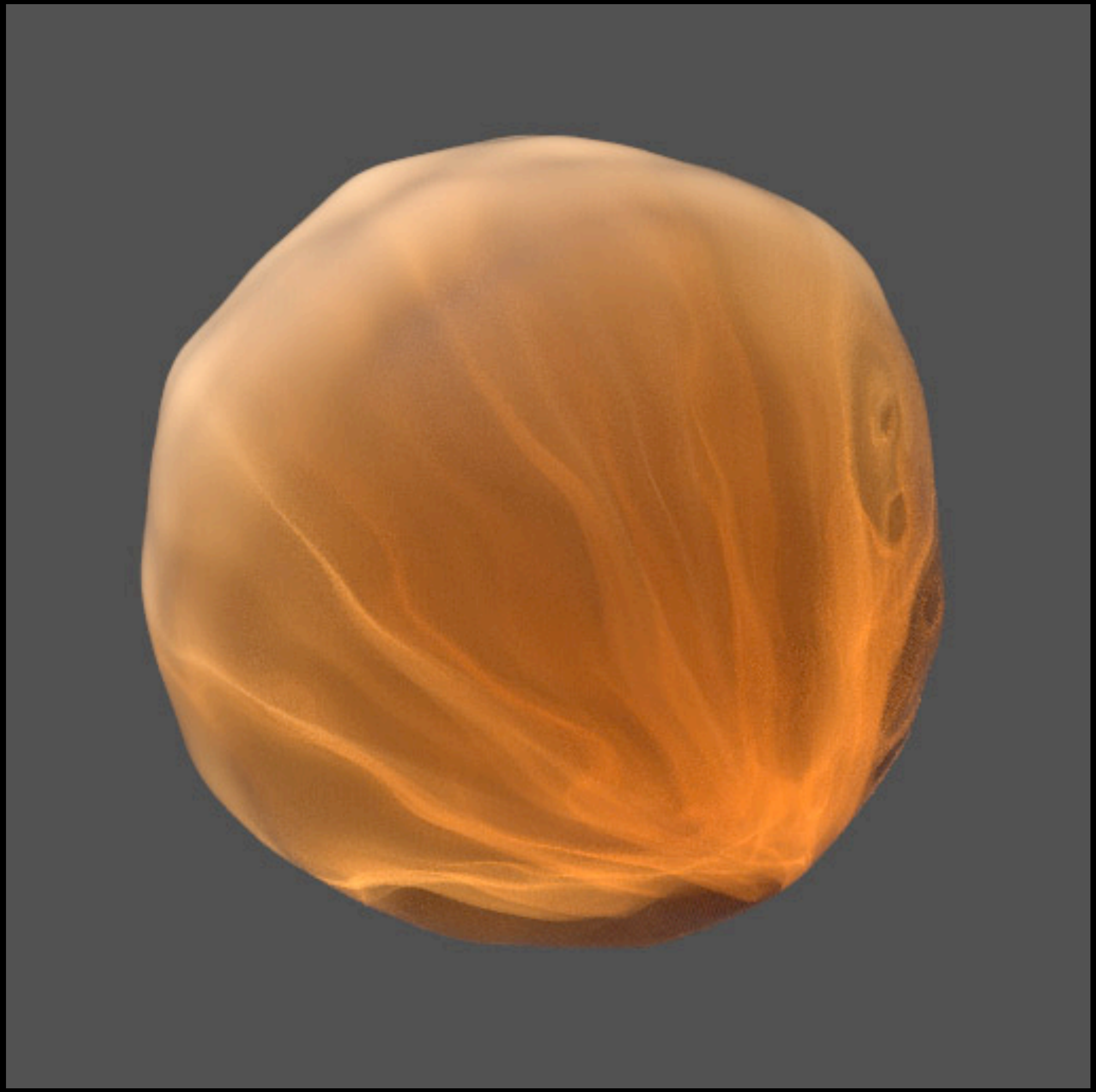
Our method

Results - Bumpy sphere

- Bumpy sphere (9680 triangles)



Photon map 10M (equal time)



Our method

Conclusion

- New method for single scatter in refractive media
  - Applicable to many rendering algorithms
  - New half-vector formulation
  - Efficient culling and search methods
  - Supports shading normals and large triangle meshes
  - Interactive performance for some scenes
- Future work
  - Better culling
  - Reflections and low-order scattering
  - Multiple interfaces

Acknowledgements

- Sponsors
  - NSF
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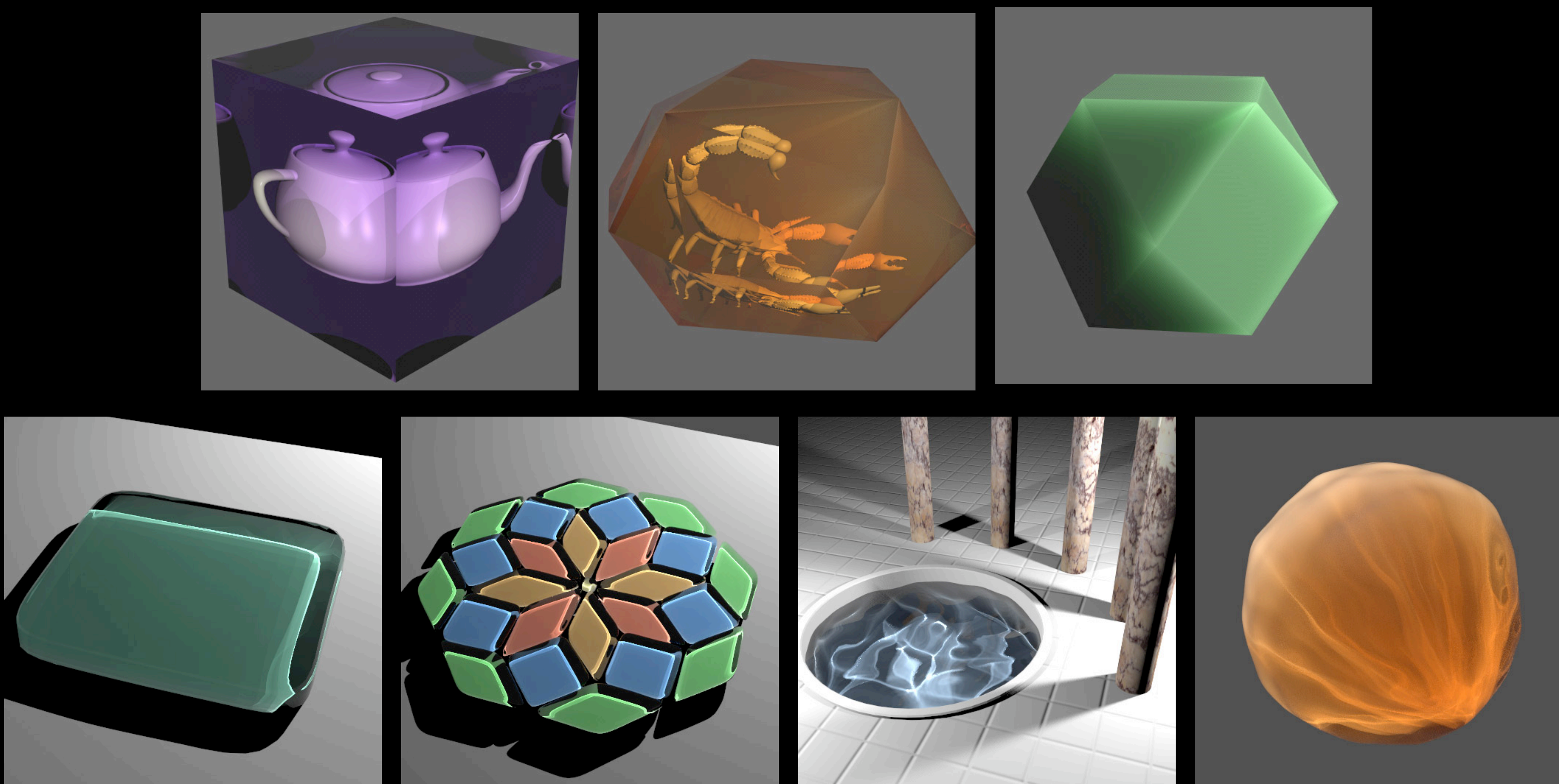
  - Intel
  - NVidia
  - Microsoft
  - INRIA sabbatical program
- PCG Graphics Lab and Elizabeth Popolo

The End



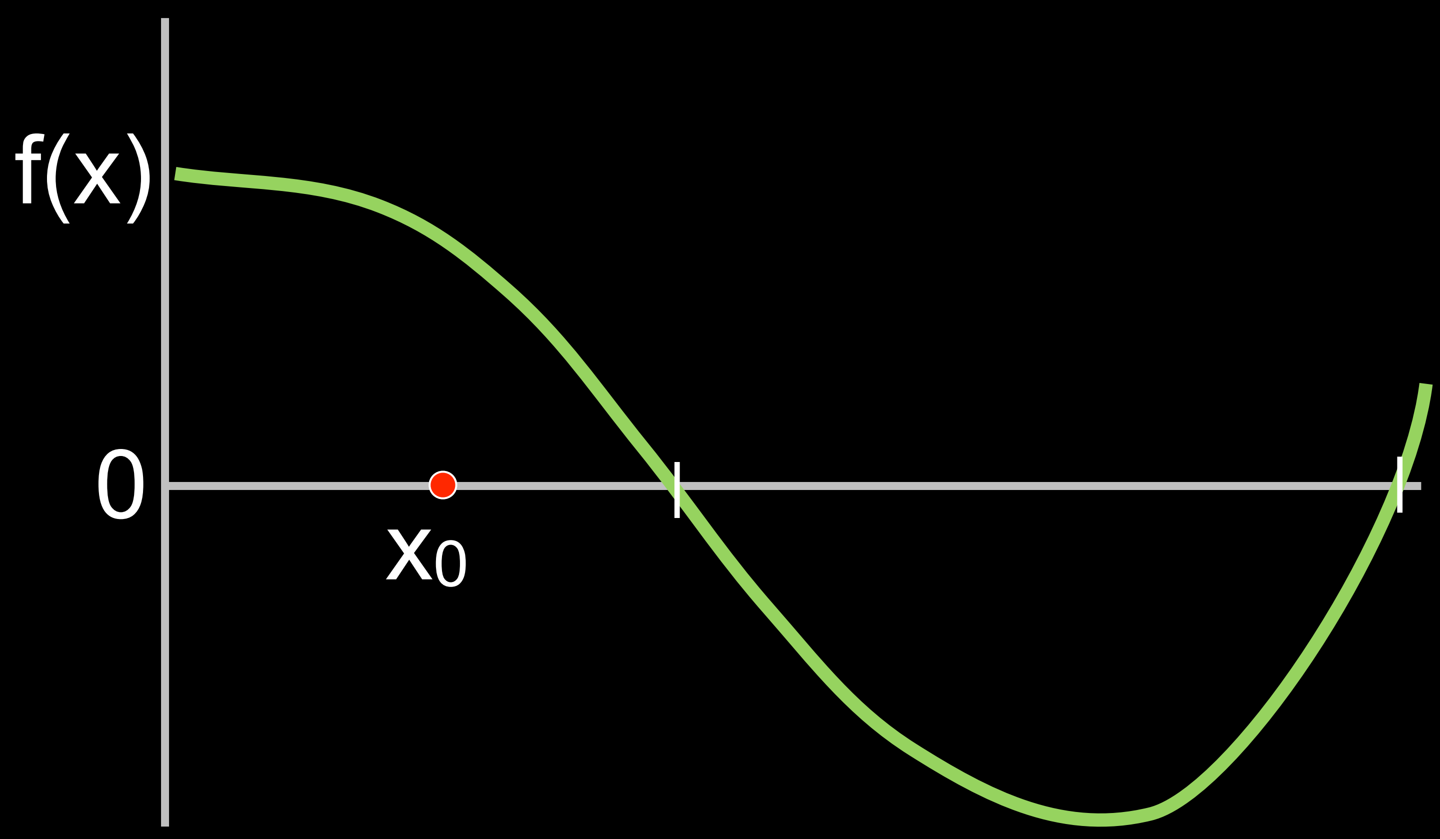
Results - CPU timings

Name	Render Time	Triangles		Shading Normals
		Surface	Other	
Teapot	15.3 s	12	4096	No
Cuboctahedron	13.9 s	20	0	No
Amber	19.2 s	36	60556	No
Glass tile	66.9 s	798	60	Yes
Glass mosaic	87.8 s	20813	1450	Yes
Pool	59.4 s	2632	4324	Yes
Bumpy Sphere	304.3 s	9680	0	Yes

512x512 images, 64 samples per pixel (128 for bumpy sphere), 8-core 2.83GHz Intel Core2 CPU

Newton's Method

- Iterative root finding method
  - Start with initial guess x0
  - Iteration: $x_{i+1} = x_i - f(x_i) / f'(x_i)$



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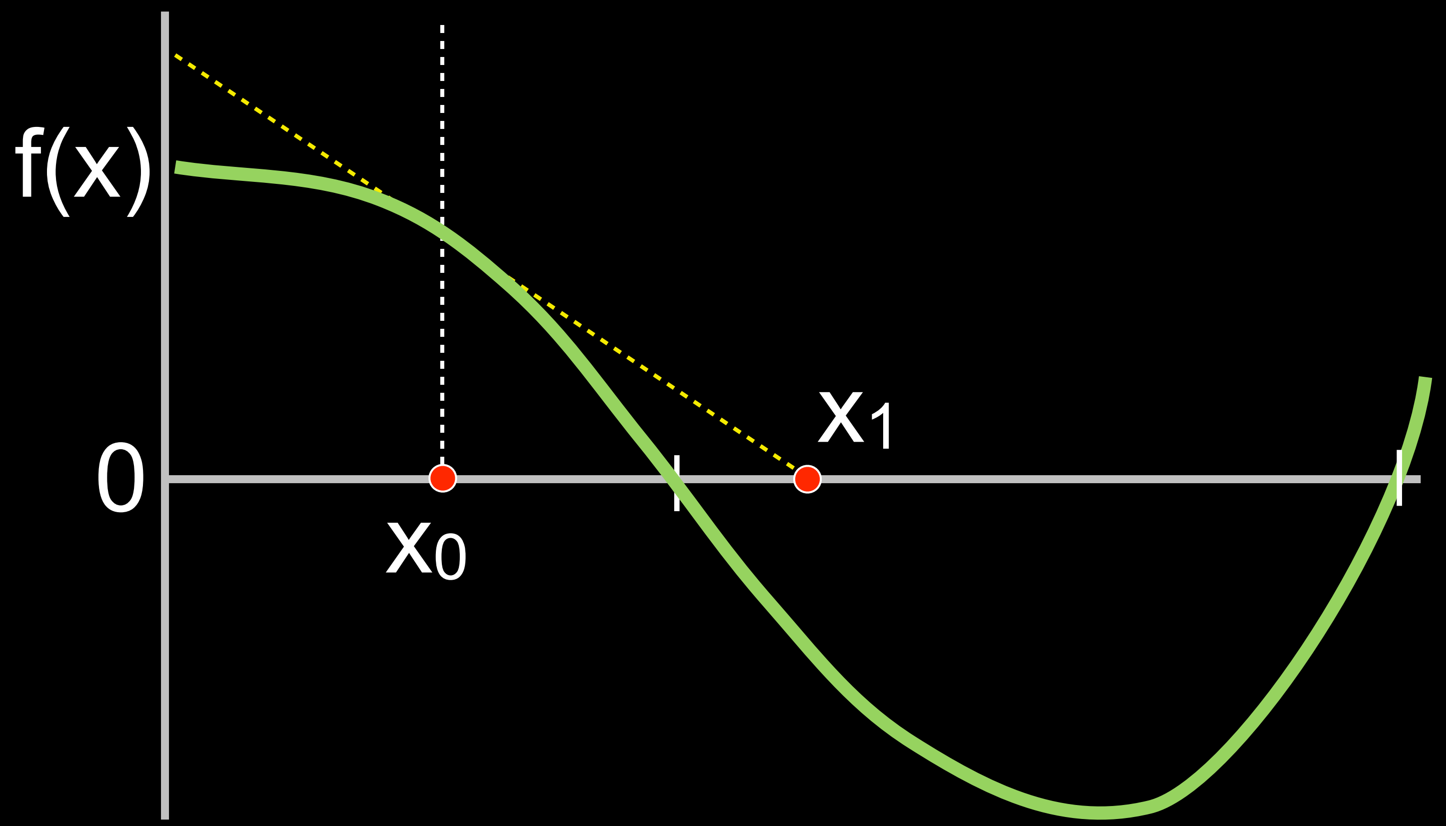

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